

Math 124 Quiz #7 Solutions

1. (a) Here are a couple of ways to take the derivative:

- Using the chain and product rule on the first term and the chain rule on the second term:

$$f'(t) = \frac{1}{t \cdot e^t}(e^t + t \cdot e^t) + \frac{3}{1+(4t)^2} \cdot 4$$

- Using logarithmic rules to simplify $f(t)$:
$$f(t) = \ln(t) + \ln(e^t) + 3 \arctan(4t)$$
$$= \ln(t) + t + 3 \arctan(4t)$$

$$\text{So, } f'(t) = \frac{1}{t} + 1 + \frac{3}{1+(4t)^2} \cdot 4.$$

(b) To get the derivative, I will use logarithmic differentiation because this is not an exponential function (the base is not constant) and I cannot use the power rule (the exponent is not constant).

Taking the natural log of both sides: $\ln(y) = \sin(x) \cdot \ln(x)$

Taking the derivative of both sides: $\frac{d}{dx}[\ln(y)] = \frac{d}{dx}[\sin(x) \cdot \ln(x)]$
 $\frac{1}{y} \cdot \frac{dy}{dx} = \cos(x) \cdot \ln(x) + \frac{\sin(x)}{x}$

$$\frac{dy}{dx} = y[\cos(x) \cdot \ln(x) + \frac{\sin(x)}{x}]$$

$$\frac{dy}{dx} = x^{\sin(x)}[\cos(x) \cdot \ln(x) + \frac{\sin(x)}{x}]$$

2. (a) The velocity is given by $v = \frac{ds}{dt} = 3t^2 - 12t + 9$.

The velocity at 2 seconds is $v = 3(2)^2 - 12(2) + 9 = -3$ feet/sec.

So, the speed is $|-3|$ feet/sec = 3 feet/sec.

(b) The swallow is at rest when $v = 0 \Rightarrow 3t^2 - 12t + 9 = 0$
 $\Rightarrow 3(t-3)(t-1) = 0$
 $\Rightarrow t = 1 \text{ or } t = 3 \text{ seconds}$

The swallow is moving in the positive direction when $v > 0$.

Here are a couple of ways to determine where $v > 0$:

- If $0 \leq t < 1$, then both of the factors $t - 1$ and $t - 3$ are negative, so $v > 0$.
If $1 < t < 3$, then the factor $t - 1$ is positive, but the factor $t - 3$ is negative, so $v < 0$.
If $t > 3$, then both of the factors $t - 1$ and $t - 3$ are positive, so $v > 0$.
- Note that the graph of $v = 3t^2 - 12t + 9$ is a parabola that opens upward. Since it has zeros at $t = 1$ and $t = 3$, it is positive for $0 \leq t < 1$ and $t > 3$.

So the swallow is at rest at 1 or 3 seconds and it is moving in the positive direction when $0 \leq t < 1$ and $t > 3$.