Math 124 Quiz #6 Solutions

1. (a) Using the chain rule, we have that $\frac{dy}{dx} = 6^{\tan x} \cdot \ln(6) \cdot \frac{d}{dx}[\tan x]$

$$= 6^{\tan x} \cdot \ln(6) \cdot \sec^2 x.$$

(b) Using the chain rule on the first term and the product and chain rule (in that order) on the second term:

$$f'(t) = \sec(\sqrt{t})\tan(\sqrt{t}) \cdot \tfrac{d}{dt}[\sqrt{t}] \quad + \quad \tfrac{d}{dt}[2t] \cdot (t^3+5)^8 \quad + \quad 2t \cdot \tfrac{d}{dt}[(t^3+5)^8]$$

$$= \sec(\sqrt{t})\tan(\sqrt{t}) \cdot \tfrac{1}{2}t^{-1/2} \quad + \quad 2(t^3+5)^8 \quad + \quad 2t \cdot 8(t^3+5)^7 \cdot 3t^2$$

2. $$\lim_{x\to\pi} \frac{\tan x}{2\sin x} = \lim_{x\to\pi} \frac{\frac{\sin x}{\cos x}}{2\sin x} = \lim_{x\to\pi} \frac{\sin x}{\cos x} \cdot \frac{1}{2\sin x}$$

$$= \lim_{x\to\pi} \frac{1}{2\cos x}$$

$$= \frac{1}{2\cos(\pi)}$$

$$= -\frac{1}{2}$$

3. Differentiating both sides with respect to x:

$$\frac{d}{dx}(x^3 + \sin y + y) = \frac{d}{dx}(1)$$

$$3x^2 + \cos y \tfrac{dy}{dx} + \tfrac{dy}{dx} = 0$$

Plugging in $x = 1$ and $y = 0$*:

$$3(1)^2 + \cos(0)\tfrac{dy}{dx} + \tfrac{dy}{dx} = 0$$

$$3 + 2\tfrac{dy}{dx} = 0 \qquad \Rightarrow \qquad \frac{dy}{dx} = -\frac{3}{2}.$$

(*If you solve for $\frac{dy}{dx}$, you should have $\frac{dy}{dx} = \frac{-3x^2}{\cos y + 1}$.)

Equation of the tangent line: $y = -\frac{3}{2}(x-1)$.