

Math 151 Quiz #4 Solutions

1. (a) Note that $f(x) = -\frac{2}{x^2} + 8 = -2x^{-2} + 8$.

Using the power rule, we have that $f'(x) = -2(-2)x^{-3} + 0 = 4x^{-3}$ or $f'(x) = \frac{4}{x^3}$.

$$\begin{aligned}
 \text{(b) } f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{-\frac{2}{(x+h)^2} + 8 - (-\frac{2}{x^2} + 8)}{h} = \lim_{h \rightarrow 0} \frac{-\frac{2}{(x+h)^2} + \frac{2}{x^2}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-\frac{2x^2}{x^2(x+h)^2} + \frac{2(x+h)^2}{x^2(x+h)^2}}{h} \quad (\text{Common denom.}) \\
 &= \lim_{h \rightarrow 0} \frac{\frac{-2x^2 + 2(x^2 + 2xh + h^2)}{x^2(x+h)^2}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{-2x^2 + 2x^2 + 4xh + 2h^2}{x^2(x+h)^2}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{x^2(x+h)^2} \cdot \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4x + 2h}{x^2(x+h)^2} \\
 &= \frac{4x + 2(0)}{x^2(x+0)^2} = \frac{4x}{x^4} = \frac{4}{x^3}
 \end{aligned}$$

- (c) The slope of f at $x = 1$ is given by $f'(1) = \frac{4}{1^3} = 4$.

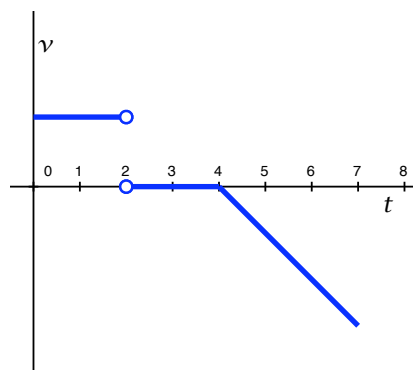
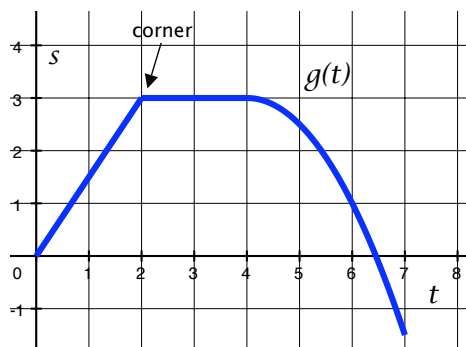
2. (a) Note that the velocity function is the derivative of the position function with respect to time.

Also, note that between 0 and 2 seconds, the llama has a constant positive velocity (graph of $g(t)$ has a constant positive slope).

The velocity at 2 seconds is undefined, since $g(t)$ has a corner at $t = 2$.

The velocity between time 2 and time 4 is 0 meters/second since the slope of $g(t)$ is 0 between 2 and 4 seconds.

The velocity is negative from time 4 to time 7 since the slope of $g(t)$ is negative on that interval. As $t \rightarrow 7$, the slope of $g(t)$ becomes more negative, so the velocity graph decreases as $t \rightarrow 7$.



(Note: Your graph of velocity may look slightly different. Make sure it satisfies the criteria above.)

(b) The velocity at time 1 will be the slope of the function $g(t)$ at $t = 1$ or $g'(1)$.

The curve is linear between time 0 and time 2, so the slope of the curve can be computed using 2 points on the curve with x -values between or including 0 and 2. For example, we can calculate the slope using the points $(0,0)$ and $(2,3)$.

The velocity at time 1 is given by $\frac{3-0}{2-0} = \frac{3}{2} = 1.5$ meters/second.