Math 151 Quiz #3 Solutions

1. Note that $f(x) = \frac{3x+5}{x^2-x-6} = \frac{3x+5}{(x-3)(x+2)}$ (Factored as much as possible.)

   From this, we can see that $f(x)$ has the vertical asymptotes $x = 3$ and $x = -2$.

   To find the horizontal asymptotes, consider that $\lim_{x\to\infty} \frac{3x+5}{x^2-x-6} = 0$ and $\lim_{x\to-\infty} \frac{3x+5}{x^2-x-6} = 0$.

   Thus, $f(x)$ has the horizontal asymptote $y = 0$.

2. (a) $\lim_{t\to\ -\infty} \frac{-t^5+8}{4t^5-10t^2+8} = -\frac{1}{4}$ since the degree in the numerator is equal to the degree in the denominator.

   (b) $\lim_{x\to-2} \frac{x+3}{\sqrt{\sin(\pi x)+x^2}} = \frac{-2+3}{\sqrt{\sin(-2\pi)+(-2)^2}} = \frac{1}{\sqrt{0+4}} = \frac{1}{2}$

3. We can use either definition.

   - Slope of the tangent at $x = 2$ $= f'(2) = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2} \frac{3x^2+x-(3(2)^2+2)}{x-2}$

$$= \lim_{x\to 2} \frac{3x^2+x-14}{x-2}$$

$$= \lim_{x\to 2} \frac{(3x+7)(x-2)}{x-2}$$

$$= \lim_{x\to 2} 3x+7$$

$$= 3(2)+7$$

$$= 13$$

   Slope of the tangent at $x = 2$ $= f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{3(2+h)^2+2+h-(3(2)^2+2)}{h}$

$$= \lim_{h\to 0} \frac{3(4+4h+h^2)+2+h-14)}{h}$$

$$= \lim_{h\to 0} \frac{13h+3h^2}{h}$$

$$= \lim_{h\to 0} 13+3h$$

$$= 13+3(0)$$

$$= 13$$

   So, the slope of the tangent is 13.