Math 151 Quiz #10 Solutions

1. To find the critical numbers, we need to find x-values where the derivative $g'(x)$ is equal to 0 or is undefined.

$$g'(x) = e^{x^3-6x} \cdot (3x^2 - 6) \qquad \text{(Using the chain rule.)}$$

This is defined for all real values, so we can focus on values for which $e^{x^3-6x} \cdot (3x^2 - 6) = 0$.

Since e^{x^3-6x} is never zero (In fact, it is always positive.), we need $3x^2 - 6 = 0 \quad \Rightarrow \quad x^2 = 2 \quad \Rightarrow \quad x = \pm\sqrt{2}$.

So, the critical numbers are $x = \pm\sqrt{2}$.

2. **Critical Numbers:**
First, we need to find the critical numbers in the domain. $f'(x) = \frac{(x^2+4)(4) - 4x(2x)}{(x^2+4)^2}$ (Quotient Rule)
$= \frac{-4x^2+16}{(x^2+4)^2}$

Note that this is defined for all x-values since $x^2 + 4$ is always positive (never zero).

So, we need to find values for which $\frac{-4x^2+16}{(x^2+4)^2} = 0. \quad \Rightarrow \quad -4x^2 + 16 = 0 \quad \Rightarrow \quad x^2 = 4 \quad \Rightarrow \quad x = \pm 2$

Note that in our domain, the only critical number is $x = 2$.

Evaluating $f(x)$ at the endpoints and the critical numbers:

$f(0) = 0 \qquad f(2) = \frac{8}{8} = 1 \qquad f(6) = \frac{24}{40} = \frac{3}{5}$

Absolute Maximum Value: 1 **Absolute Minimum Value:** 0

3. (a) The function $f(x)$ is increasing when $f'(x) > 0$ (when the graph of $f'(x)$ is above the x-axis).

$\Rightarrow \quad f(x)$ is increasing for $x < -2, \quad 1 < x < 3.5, \quad x > 3.5$

(b) The function $f(x)$ will have a local maximum when $f'(x)$ switches from positive to negative.

$\Rightarrow \quad f(x)$ has a local maximum at $x = -2$.

(c) The function $f(x)$ will be concave up when the second derivative $f''(x) > 0 \quad \Rightarrow$ The slope of $f'(x)$ is positive.

$\Rightarrow \quad f(x)$ is concave up for $-1 < x < 2, \quad x > 3.5$