Math 124 Worksheet #6 Solutions

1. Differentiate the following.

   (a) $y = \frac{1}{2}\sec x \tan x$

   Using the product rule:
   $$\frac{dy}{dx} = \frac{1}{2}\left[\frac{d}{dx}(\sec x)\tan x + \sec x \cdot \frac{d}{dx}(\tan x)\right]$$
   $$= \frac{1}{2}[(\sec x \tan x)\tan x + \sec x(\sec^2 x)]$$
   $$= \frac{1}{2}[\sec x \tan^2 x + \sec^3 x]$$

   (b) $f(\theta) = \dfrac{e^\theta}{\cos \theta} - \cot x$

   Using the quotient rule on the first term:
   $$f'(\theta) = \frac{\cos \theta \frac{d}{d\theta}(e^\theta) - e^\theta \frac{d}{d\theta}(\cos \theta)}{(\cos \theta)^2} - (-\csc^2\theta)$$
   $$= \frac{e^\theta \cos \theta - e^\theta(-\sin \theta)}{\cos^2\theta} + \csc^2\theta$$
   $$= \frac{e^\theta \cos \theta + e^\theta \sin \theta}{\cos^2\theta} + \csc^2\theta$$

2. $f(\theta) = \sqrt{3}\cos \theta - \sin \theta$

   (a) Find the roots/zeros of the function f.

   We want to find the θ-values for which $f(\theta) = 0$.
   $$\Rightarrow \quad \sqrt{3}\cos \theta - \sin \theta = 0$$
   $$\sqrt{3}\cos \theta = \sin \theta$$
   $$\sqrt{3} = \tan \theta \quad \text{(Div. by } \cos \theta\text{)}$$

   Note: $\arctan(\sqrt{3}) = \frac{\pi}{3}$, so $\tan \theta = \sqrt{3}$ when $\theta = \frac{\pi}{3} + k\pi$ for an integer k

   Thus, the roots of f are $\theta = \frac{\pi}{3} + k\pi$ for an integer k.

   (b) For what values of θ does f have a horizontal tangent?
   The function f will have a horizontal tangent when $f'(\theta) = 0$.
   $$\Rightarrow \quad -\sqrt{3}\sin \theta - \cos \theta = 0$$
   $$-\sqrt{3}\sin \theta = \cos \theta$$
   $$-\sqrt{3}\tan \theta = 1$$
   $$\tan \theta = -\frac{1}{\sqrt{3}}$$

   Note: $\arctan(-\frac{1}{\sqrt{3}}) = -\frac{\pi}{6}$, so $\tan \theta = -\frac{1}{\sqrt{3}}$ when $\theta = -\frac{\pi}{6} + k\pi$ for an integer k

   Thus, f has horizontal tangents at the values $\theta = -\frac{\pi}{6} + k\pi$ for an integer k.

(c) What are the maximum and minimum values of the function? (Consider the graph of the function.)

Considering the graph of the function, we can see that the maximum and minimum values of f will occur when f has a horizontal tangent. So, f has a maximum or minimum when $\theta = -\frac{\pi}{6} + k\pi$ for an integer k. Plugging the values of θ into the original function f will yield either 2 or -2, so the maximum value of f is 2 and the minimum value is -2.