Math 124 Worksheet #5 Solutions

1. Find an equation of the tangent line to $y = \dfrac{\sqrt{x}}{x}$ at $x = 4$.

   The y-coordinate of the point at which the line is tangent is $y = \dfrac{\sqrt{4}}{4} = \frac{1}{2}$. So, a point on the tangent line is $(4, \frac{1}{2})$.

   Note that $y = \dfrac{\sqrt{x}}{x} = \dfrac{x^{1/2}}{x^1} = x^{-1/2}$. $\Rightarrow$ $\dfrac{dy}{dx} = -\frac{1}{2}x^{-3/2} = -\dfrac{1}{2\sqrt{x^3}}$

   So, the slope of the tangent line is $\dfrac{dy}{dx}|_{x=4} = -\dfrac{1}{2\sqrt{4^3}} = -\dfrac{1}{16}$.

   Tangent Line Equation: $y - \frac{1}{2} = -\frac{1}{16}(x-4)$ or $y = -\frac{1}{16}x + \frac{3}{4}$

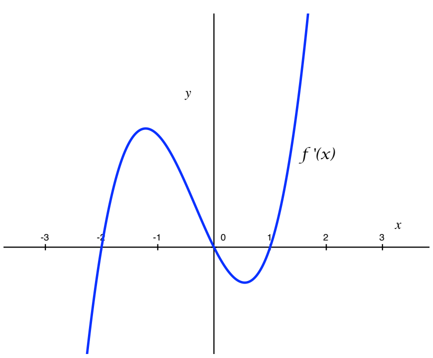
2. Find $h'''(x)$ if $h(x) = 2x^3 - 5x^2 - x + 82 + 9e^x$.

   1st Derivative: $h'(x) = 2(3x^2) - 5(2x) - 1 + 0 + 9e^x$
   $= 6x^2 - 10x - 1 + 9e^x$

   2nd Derivative: $h''(x) = 6(2x) - 10 + 0 + 9e^x$
   $= 12x - 10 + 9e^x$

   3rd Derivative: $h'''(x) = 12 + 0 + 9e^x$
   $= 12 + 9e^x$

3. The following is a graph of $f'(x)$, the derivative of $f(x)$.



   (a) For what x-values does $f(x)$ have a horizontal tangent?

   If f has a horizontal tangent at a value x, then $f'(x)$ must equal zero. So, f has horizontal tangents at $x = -2$, $x = 0$, and $x = 1$.

   (b) For what x-values does $f(x)$ have positive slope?

   The derivative $f'(x)$ will be positive when $f(x)$ has positive slope. So, $f(x)$ has positive slope for $-2 < x < 0$ and $x > 1$ (Intervals: $(-2, 0) \cup (1, \infty)$).

   True or False: $f(-1) > f(0)$

   FALSE: Given the information in part (b), we have that f has positive slope between $x = -2$ and $x = 0$. So, the function f is increasing (rising from left to right). So, the values of f are increasing between $x = -2$ and $x = 0$ $\Rightarrow$ $f(0)$ will be greater than $f(-1)$.

For what x-values is $f''(x) \leq 0$?

Note that $f''(x)$ = the slope of f' at a value x. So, $f''(x) \leq 0$ for values of x for which f' has negative or 0 slope $\quad \Rightarrow \quad -1 \leq x \leq 0.4$ (approximately).

4. Suppose the position of a moose (in kilometers) from a lake is given by $s = f(t) = 2t^4 - 3t^2 + t$ in hours. What is the velocity of the moose at 2 hours? Is the moose's velocity increasing or decreasing at 2 hours? (Hint: What is the moose's acceleration at 2 hours?)

   The velocity of the moose at time t is $v = f'(t) = 2(4t^3) - 3(2t) + 1 = 8t^3 - 6t + 1$.

   $\Rightarrow$ Velocity at 2 hours $= f'(2) = 8(2)^3 - 6(2) + 1 = 53$ km/hr

   (That's a fast moose!!)

   The acceleration of the moose at time t is $a = f''(t) = 8(3t^2) - 6 + 0 = 24t^2 - 6$.

   $\Rightarrow$ Acceleration at 2 hours $= f''(2) = 24(2)^2 - 6 = 90$ km/hr^2

   Since the acceleration is positive, the velocity is increasing at 2 hours. (Acceleration is the rate of change of velocity.)