

Math 124 Worksheet #4 Solutions

1. Let $f(x) = \sqrt{3x - 5}$

(a) Find $f'(x)$. For what values is the derivative defined?

You can use either definition of the derivative:

$$\begin{aligned} \bullet f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (\text{provided the limit exists}) \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{3(x+h) - 5} - \sqrt{3x - 5}}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x+h) - 5 - (3x - 5)}{h(\sqrt{3(x+h) - 5} + \sqrt{3x - 5})} \quad (\text{Mult. num. and denom. by conjugate}) \\ &= \lim_{h \rightarrow 0} \frac{3h}{h(\sqrt{3(x+h) - 5} + \sqrt{3x - 5})} \\ &= \lim_{h \rightarrow 0} \frac{3}{(\sqrt{3(x+h) - 5} + \sqrt{3x - 5})} \\ &= \frac{3}{\sqrt{3x-5} + \sqrt{3x-5}} \\ &= \frac{3}{2\sqrt{3x-5}} \end{aligned}$$

$$\begin{aligned} \bullet f'(a) &= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \quad (\text{provided the limit exists}) \\ &= \lim_{x \rightarrow a} \frac{\sqrt{3x - 5} - \sqrt{3a - 5}}{x - a} \\ &= \lim_{x \rightarrow a} \frac{3x - 5 - (3a - 5)}{(x - a)(\sqrt{3x - 5} + \sqrt{3a - 5})} \quad (\text{Mult. num. and denom. by conjugate}) \\ &= \lim_{x \rightarrow a} \frac{3x - 3a}{(x - a)(\sqrt{3x - 5} + \sqrt{3a - 5})} \\ &= \lim_{x \rightarrow a} \frac{3(x - a)}{(x - a)(\sqrt{3x - 5} + \sqrt{3a - 5})} \\ &= \lim_{x \rightarrow a} \frac{3}{\sqrt{3x - 5} + \sqrt{3a - 5}} \\ &= \frac{3}{\sqrt{3a-5} + \sqrt{3a-5}} \\ &= \frac{3}{2\sqrt{3a-5}} \end{aligned}$$

In either case, $f'(x) = \frac{3}{2\sqrt{3x-5}}$, which is defined for values $x > \frac{5}{3}$ whereas the original function f is defined for $x \geq \frac{5}{3}$.

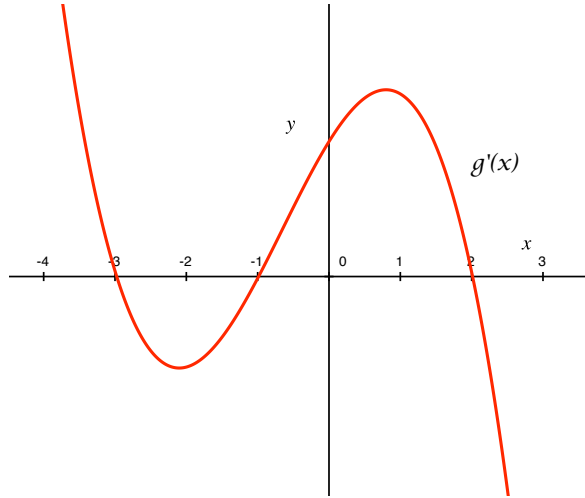
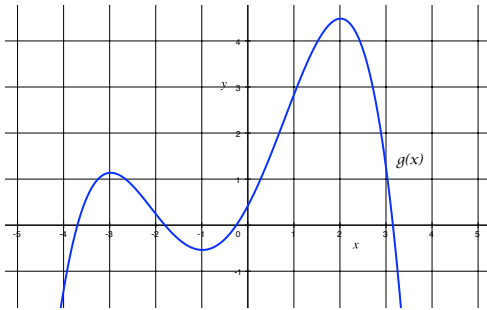
(b) Find an equation for the tangent line of f at $x = 3$.

Since $f(3) = \sqrt{3(3) - 5} = 2$, the tangent line must go through the point $(3, 2)$.

Since $f'(3) = \frac{3}{2\sqrt{3(3)-5}} = \frac{3}{4}$, the tangent line has slope $\frac{3}{4}$.

So, an equation of the tangent line is $y - 2 = \frac{3}{4}(x - 3)$ or $y = \frac{3}{4}x - \frac{1}{4}$.

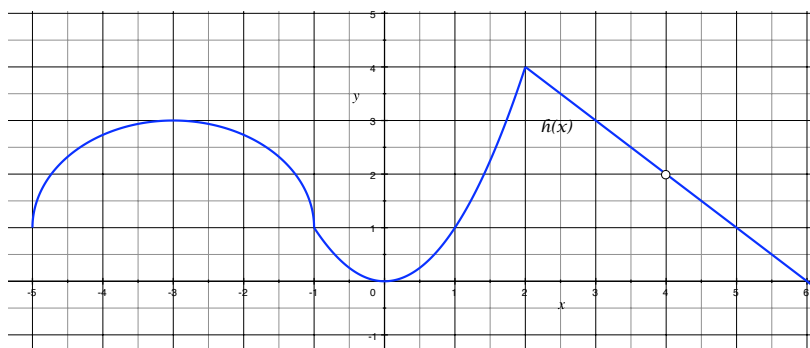
2. Given the graphs of g and h below, sketch graphs of g' and h' .

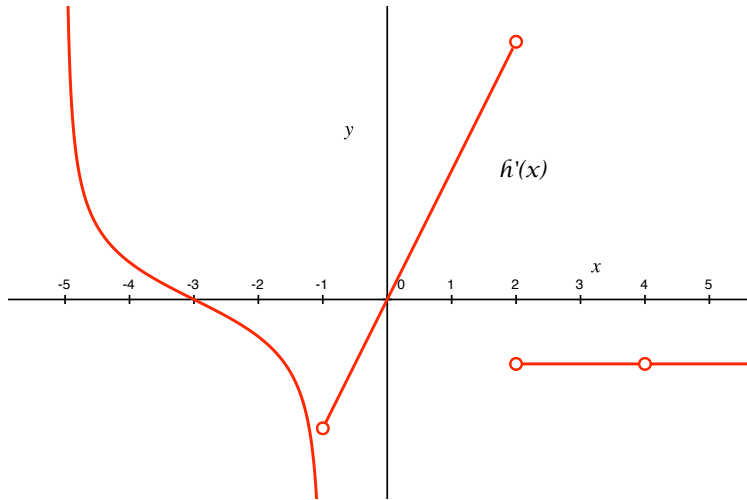


Note that for $g(x)$, we have that $g'(x) = 0$ at $x = -3$, $x = -1$, and $x = 2$ because the tangent line of g is horizontal at those values.

For $x < -3$, the slope is positive and decreases to 0 ($g'(x) > 0$). For $-3 < x < -1$, the slope is negative ($g'(x) < 0$). For $-1 < x < 2$, the slope is positive ($g'(x) > 0$). For $x > 2$, the slope is negative ($g'(x) < 0$).

The graph of $g'(x)$ above is a **possible** graph of g' .





Note that for $h(x)$, we have that $h'(x) = 0$ at $x = -3$ and $x = 0$. The function h has vertical tangent lines at $x = -5$ and $x = -1$, so we can expect vertical asymptotes at those points for h' .

For $-5 < x < -3$, the slope of h is positive ($h'(x) > 0$). For $-3 < x < -1$, the slope of h is negative ($h'(x) < 0$). For $-1 < x < 0$, the slope of h is positive ($h'(x) > 0$). For $0 < x < 2$, the slope of h is negative ($h'(x) < 0$). For $2 < x$ (except at $x = 4$) the slope of h is negative and constant.

The graph will have holes at $x = -1$, $x = 2$, and $x = 4$ due to discontinuities, kinks (or corners), and vertical asymptotes of h .

The graph of h' above is a **possible** graph of h' .