Math 124 Worksheet #3 Solutions

1. Evaluate the following limits.

   (a) $\lim_{x\to-\infty} \frac{x+1}{3x^5-2x+5}$

   $\lim_{x\to-\infty} \frac{x+1}{3x^5-2x+5} = 0$ since the degree of the numerator is less than the degree of the denominator.

   OR

   $$\lim_{x\to-\infty} \frac{x+1}{3x^5-2x+5} = \lim_{x\to-\infty} \frac{x+1}{3x^5-2x+5} \cdot \frac{\frac{1}{x^5}}{\frac{1}{x^5}} = \lim_{x\to-\infty} \frac{\frac{1}{x^4}+\frac{1}{x^5}}{3-\frac{2}{x^4}+\frac{5}{x^5}} = \frac{0}{3} = 0$$

   (b) $\lim_{x\to\infty} \frac{\sqrt{9x^2+x}}{2x-5}$

   $$\lim_{x\to\infty} \frac{\sqrt{9x^2+x}}{2x-5} = \lim_{x\to\infty} \frac{\sqrt{9x^2+x}}{2x-5} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{\frac{1}{x^2}(9x^2+x)}}{2-\frac{5}{x}}$$
   $$= \lim_{x\to\infty} \frac{\sqrt{9+\frac{1}{x}}}{2-\frac{5}{x}} = \frac{\sqrt{9}}{2} = \frac{3}{2}$$

2. Find the horizontal and vertical asymptotes of the curve $y = 2 + e^{3/x}$.

   This function will have a vertical asymptote at $x = 0$ since $\frac{3}{x}$ will approach ∞ as x approaches 0 from the right. This will make the term $e^{e/x}$ approach ∞.
   So, $\lim_{x\to0^+} [2 + e^{3/x}] = \infty$, which tells us that there is a vertical asymptote at $x = 0$.

   To find the horizontal asymptotes, we must find the limits at infinity for the function.

   Consider $\lim_{x\to\infty} [2 + e^{3/x}] = 2 + e^0 = 3$ since $\frac{3}{x}$ approaches 0 as x becomes large.

   Consider $\lim_{x\to-\infty} [2 + e^{3/x}] = 2 + e^0 = 3$ since $\frac{3}{x}$ approaches 0 as x becomes large (negatively).

   So, $y = 3$ is the horizontal asymptote of the curve and $x = 0$ is the vertical asymptote.

3. If the position of a rabid squirrel is given by $s = 15 - \frac{2}{t^2}$ in feet at t seconds, what is the velocity of the squirrel at 1 second?*

   The velocity at 1 second can be found by finding the slope of the tangent line to the position curve $s = f(t)$ at 1 second.

   You can use either definition 1 or 2:

- Velocity $= \lim_{t\to 1} \frac{f(t) - f(1)}{t-1} = \lim_{t\to 1} \frac{15 - \frac{2}{t^2} - 13}{t-1} = \lim_{t\to 1} \frac{2 - \frac{2}{t^2}}{t-1}$

$$= \lim_{t\to 1} \frac{\frac{2t^2-2}{t^2}}{t-1}$$

$$= \lim_{t\to 1} \frac{\frac{2(t+1)(t-1)}{t^2}}{t-1}$$

$$= \lim_{t\to 1} \frac{2(t+1)}{t^2} = 4 \text{ ft/sec}$$

- Velocity $= \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{15 - \frac{2}{(1+h)^2} - 13}{h} = \lim_{h\to 0} \frac{2 - \frac{2}{(1+h)^2}}{h}$

$$= \lim_{h\to 0} \frac{\frac{2(1+h)^2-2}{(1+h)^2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2(1+2h+h^2)-2}{(1+h)^2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{4h+2h^2}{(1+h)^2}}{h}$$

$$= \lim_{h\to 0} \frac{4+2h}{(1+h)^2} = 4 \text{ ft/sec}$$

*Note: You should be running at least this fast at 1 second to avoid the squirrel.