Math 124 Worksheet #2 Solutions

1. For what values is $\ln(\tan^2 x) + 2x^5$ continuous?

   Note that $2x^5$ is continuous for all real numbers (domain is all real numbers).

   Also note that $\tan^2 x = (\tan\, x)^2$. So $\tan^2 x$ is defined and continuous when $\tan\, x$ is defined and continuous. The domain for $\tan\, x$ is all real numbers except $x = \frac{\pi}{2} + k\pi$, for an integer k.

   The function $\ln(\tan^2 x)$ will be continuous whenever it is defined. The domain of natural log is all positive numbers. So, for the composition $\ln(\tan^2 x)$ to be defined, we must have $\tan^2 x > 0$. Of particular concern are the values of x for which $\tan^2 x = 0$, which are the values $x = k\pi$ for an integer k.

   So, $\ln(\tan^2 x)$ is defined for all real numbers except $x = \frac{\pi}{2} + k\pi$ (values for which $\tan^2 x$ is undefined) and $x = k\pi$ (values for which $\tan^2 x = 0$) for an integer k.

   You can write this in one statement as $\ln(\tan^2 x)$ is defined for all real numbers except $x = \frac{\pi}{2} + \frac{k\pi}{2}$ for an integer k.

2. Consider the domain of the function in the limits and evaluate the limits.

   (a) $\lim\limits_{n \to 3} \sqrt{2^n - 1}$

   - Domain: $\sqrt{2^n - 1}$ will be defined when $2^n - 1 \geq 0 \quad \Rightarrow \quad 2^n \geq 1$
     $\Rightarrow \quad n \geq 0$

     So, the domain is $n \geq 0$.
   - Since $\sqrt{2^n - 1}$ is continuous at $n = 3$, $\lim\limits_{n \to 3} \sqrt{2^n - 1} = \sqrt{2^3 - 1} = \sqrt{7}$.

   (b) $\lim\limits_{t \to 1^-} [1 + \arcsin\, t]$

   - Domain: $\arcsin\, t$ is defined for $-1 \leq t \leq 1$ so $1 + \arcsin\, t$ is defined for $-1 \leq t \leq 1$.
   - Since $1 + \arcsin\, t$ is continuous at 1, $\lim\limits_{t \to 1^-} [1 + \arcsin\, t] = 1 + \arcsin\, 1$
     $= 1 + \frac{\pi}{2}$

   (c) $\lim\limits_{\theta \to \pi/2^-} \sec\, \theta$

   - Domain: Note that $\sec\, \theta = \dfrac{1}{\cos\, \theta}$. So, $\sec\, \theta$ is undefined when $\cos\, \theta = 0$, which is when $\theta = \frac{\pi}{2} + k\pi$ for an integer k.

- Since $\sec\theta$ is not continuous at $\theta = \frac{\pi}{2}$, we cannot evaluate the function at $\frac{\pi}{2}$ to find the limit.
  Note that $\cos\theta$ is positive and approaching 0 as θ approaches $\frac{\pi}{2}$ with $\theta < \frac{\pi}{2}$.

$$\Rightarrow \quad \lim_{\theta\to\pi/2^-} \sec\theta = \lim_{\theta\to\pi/2^-} \frac{1}{\cos\theta} = \infty$$

(d) $\displaystyle\lim_{x\to\infty} \frac{8}{x^3-1}$

- Domain: $\frac{8}{x^3-1}$ is undefined when $x^3 = 1 \quad \Rightarrow \quad x = 1$. So, the domain is all real numbers except $x = 1$.
- As $x \to \infty$, $x^3 - 1 \to \infty$. So, $\displaystyle\lim_{x\to\infty} \frac{8}{x^3-1} = 0$ (Dividing 8 by larger and larger numbers.)

3. For what value of c is the function below continuous at $x = 0$?

$$f(x) = \begin{cases} ce^{x^2-x} & \text{if } x \leq 0 \\ 2x^2 + 1 + 2c & \text{if } x > 0 \end{cases}$$

For this function to be continuous at $x = 0$, we must have that the limit as x approaches 0 exists. So, the left and right-hand limits must equal.

$$\begin{aligned} \lim_{x\to 0^-} f(x) &= \lim_{x\to 0^+} f(x) \\ \lim_{x\to 0^-} ce^{x^2-x} &= \lim_{x\to 0^+} 2x^2 + 1 + 2c \\ ce^{0^2-0} &= 2(0)^2 + 1 + 2c \\ c &= 1 + 2c \\ c &= -1 \end{aligned}$$

So, c must equal -1 for the function $f(x)$ to be continuous.