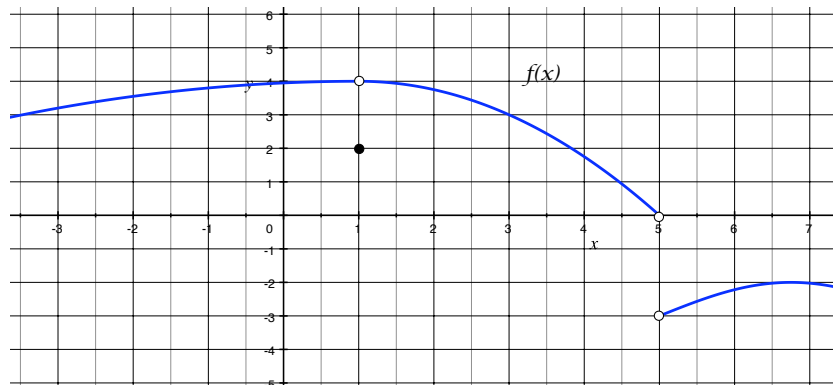


## Math 124 Worksheet #1 Solutions

1. Sketch a possible graph of a function  $f$  that satisfies the following conditions.

$$f(1) = 2, \quad \lim_{x \rightarrow 1} f(x) = 4, \quad \lim_{x \rightarrow 5^-} f(x) = 0, \quad \lim_{x \rightarrow 5^+} f(x) = -3$$

Here is a possible graph that satisfies the conditions. (There are an infinite number of possible graphs.)



2. Evaluate the following.

(a)  $\lim_{t \rightarrow 1^+} \frac{3}{1-t}$

As  $t$  approaches 1 from the right,  $1-t$  will approach 0 and since we are considering values such that  $t > 1$ ,  $1-t$  will be negative. So, as  $t$  approaches 1 from the right, we are dividing 3 by small **negative** numbers, which produces large **negative** numbers.  $\Rightarrow \lim_{t \rightarrow 1^+} \frac{3}{1-t} = -\infty$

(b)  $\lim_{t \rightarrow 1^-} \frac{3}{1-t}$

As  $t$  approaches 1 from the left,  $1-t$  will approach 0 and since we are considering values such that  $t < 1$ ,  $1-t$  will be positive. So, as  $t$  approaches 1 from the left, we are dividing 3 by small **positive** numbers, which produces large **positive** numbers.  $\Rightarrow \lim_{t \rightarrow 1^-} \frac{3}{1-t} = \infty$

(c)  $\lim_{x \rightarrow \sqrt{2}} 2x^6 - 3$

Since  $2x^6 - 3$  is a polynomial, we can evaluate the limit by plugging the value  $x = \sqrt{2}$  directly into the expression.  $\Rightarrow \lim_{x \rightarrow \sqrt{2}} 2x^6 - 3 = 2(\sqrt{2})^6 - 3 = 2(8) - 3 = 13$ .

(d)  $\lim_{t \rightarrow 1} (t^2 + 1)(3t^3 - 4t + 2)$

Recall: The limit of a product is the product of the limits, provided that each limit exists.

$$\text{So, } \lim_{t \rightarrow 1} (t^2 + 1)(3t^3 - 4t + 2) = \lim_{t \rightarrow 1} (t^2 + 1) \cdot \lim_{t \rightarrow 1} (3t^3 - 4t + 2) = 2 \cdot 1 = 2.$$

$$(e) \lim_{x \rightarrow -1} \frac{x-3}{3x^2+3}$$

Since the rational expression is defined at  $x = -1$ , we can evaluate the limit by plugging the value  $x = -1$  directly into the expression.

$$\text{So, } \lim_{x \rightarrow -1} \frac{x-3}{3x^2+3} = \frac{-1-3}{3(-1)^2+3} = \frac{-4}{6} = -\frac{2}{3}.$$

$$(f) \lim_{t \rightarrow 3} \frac{2t^2+2t-24}{t^2-3t}$$

We cannot plug  $t = 3$  directly into the expression to evaluate the limit since the expression is undefined for that value (denominator = 0).

$$\text{Note that } \frac{2t^2+2t-24}{t^2-3t} = \frac{2(t+4)(t-3)}{t(t-3)} = \frac{2(t+4)}{t} \text{ for } t \neq 0, t \neq 3.$$

$$\text{So, } \lim_{t \rightarrow 3} \frac{2t^2+2t-24}{t^2-3t} = \lim_{t \rightarrow 3} \frac{2(t+4)}{t} = \frac{2(3+4)}{3} = \frac{14}{3}.$$

$$(g) \lim_{x \rightarrow 0^-} \frac{1}{|x|}$$

As  $x$  approaches 0 from the left,  $|x|$  will approach 0 and be positive. So, as  $x$  approaches 0 from the left, we are dividing 1 by small positive numbers.

$$\Rightarrow \lim_{x \rightarrow 0^-} \frac{1}{|x|} = \infty.$$

$$(h) \lim_{x \rightarrow 0} \frac{1}{|x|}$$

Note that by similar argument to part (g),  $\lim_{x \rightarrow 0^+} \frac{1}{|x|} = \infty$ . Since the function  $\frac{1}{|x|}$

grows without bound from the left and right, we have that  $\lim_{x \rightarrow 0} \frac{1}{|x|} = \infty$ .