Math 124 Worksheet #14 Solutions

1. Find the local max and min values for the function $f(x) = \frac{x-1}{x^2}$.

   Note that the domain of f is all real numbers except $x = 0$.

   First Derivative:
   $$f'(x) = \frac{x^2(1)-(x-1)(2x)}{x^4} = \frac{-x^2+2x}{x^4} = \frac{2-x}{x^3}$$

   (Note: You can also rewrite it & use the product rule to find $f'(x)$.)

   The derivative is undefined at $x = 0$ and it is equal to zero at $x = 2$.

   Using the first or second derivative test for the critical number $x = 2$:

   - First Derivative Test:

     Sign chart for f':

f'	$- \; - \; -$	Undefined	$+ \; + \; +$	0	$- \; - \; -$
x		0		2	

     Using the first derivative test, we can see that there is a local max at $x = 2$ and the local max value is $f(2) = \frac{1}{4}$.

   - Second Derivative test:
     $$f''(x) = \frac{x^3(-1)-(2-x)(3x^2)}{x^6} = \frac{2x^3-6x^2}{x^6} = \frac{2(x-3)}{x^4}$$

     Since $f''(2) < 0$, we have a local max at $x = 2$ with a value of $f(2) = \frac{1}{4}$.

   Note: There is no local minimum for this function. Looking at the first derivative, the only possible location for a local minimum is at $x = 0$, but the function f is undefined there.

2. Sketch a graph of $f(x) = \frac{x-1}{x^2}$ by considering the following: Domain, Intercepts, Symmetry, Asymptotes, Increasing or Decreasing, Local Max/Min, Concavity, and Inflection Points.

   This will be done in class on November 28th.