

Math 124 Worksheet #11 Solutions

1. $f(x) = 1 - 3xe^{\frac{1}{4}x}$

- (a) Find the linearization of f at $x = 0$.

Note that $f(0) = 1 - 3(0)e^0 = 1$.

Derivative: $f'(x) = -3e^{\frac{1}{4}x} - 3xe^{\frac{1}{4}x} \cdot \frac{1}{4}$ (Product Rule)

$$\Rightarrow f'(0) = -3e^0 - 3(0)e^0 \cdot \frac{1}{4} = -3$$

So, the linearization of f at $x = 0$ is $L(x) = f'(0)(x - 0) + f(0)$

$$\begin{aligned} &= -3(x - 0) + 1 \\ &= -3x + 1 \end{aligned}$$

Near $x = 0$, $f(x) \approx L(x) = -3x + 1$.

- (b) Use the linearization to approximate the x -value for which $f(x) = 2$.
(Trying to algebraically solve for x such that $f(x) = 2$ is difficult, so this is a good time for approximation.)

If $f(x) = 2$, then $L(x) \approx 2 \Rightarrow 2 = 1 - 3x$
 $x = -\frac{1}{3}$

So, $f(x) = 2$ for $x \approx -\frac{1}{3}$.

(This is a fairly close approximation.)

Actual Value of f at $x = -\frac{1}{3}$: $f(-\frac{1}{3}) \approx 1.92$

2. Let $D(t)$ = the number of minutes of daylight on day t of the year in 2007. November 6th (today) is the 310th day of the year.

- (a) Interpret the following statements: $D(310) = 583.73$ and $D'(310) = -2.93$.

$D(310) = 583.73$: This means that on the 310th day (Nov. 6th) there is 583.73 minutes of daylight.

$D'(310) = -2.93$: This means that the rate of change of the minutes of daylight with respect to the day is -2.93 minutes/day. Another way to interpret this is that there will be approximately 2.93 less minutes of daylight on November 7th.

- (b) Estimate the amount of daylight on November 5th and November 10th.

Here's a tangent line approximation of the function $D(t)$ at $t = 310$:

$$D(t) \approx D'(310)(t - 310) + D(310)$$

$$= -2.93(t - 310) + 583.73$$

The minutes of daylight on November 5th is given by

$$D(309) \approx -2.93(309 - 310) + 583.73 = 2.93 + 583.73 = 586.66$$

So, there should be approximately 586.66 minutes of daylight on November 5th, which is 2.93 more minutes as on November 6th.

The minutes of daylight on November 10th is given by

$$D(314) \approx -2.93(314 - 310) + 583.73 = -2.93(4) + 583.73 = 572.01$$

So, there should be approximately 572.01 minutes of daylight on November 10th, which is $2.93(4) = 11.72$ less minutes as on November 6th.