# Math 124 Worksheet #10 Solutions

1. Suppose a forest fire spreads in a circle with radius increasing at a rate of 5 feet per minute. When the radius reaches 200 feet, at what rate is the area of the burning region increasing?

   Quantities that change with time: $A$ = Area of burning region
   $r$ = Radius of burning region

   Rates of Change: $\frac{dA}{dt} = ?$ when $r = 200$ feet
   $\frac{dr}{dt} = 5$ ft/min

   Equation Relating $A$ and $r$: $A = \pi r^2$

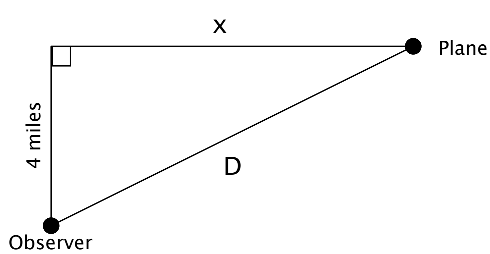
   Differentiation with respect to time:
   $$\frac{d}{dt}[A] = \frac{d}{dt}[\pi r^2]$$
   $$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

   Finding $\frac{dA}{dt}$ when $r = 200$: $\frac{dA}{dt} = 2\pi(200)(5) = 2000\pi \approx 6283.1853 \text{ ft}^2/\text{min}$

   The area is increasing by $2000\pi$ ft$^2$/min when the radius is 200 feet.

2. A plane is flying at an elevation of 4 miles over an observer. If the plane is flying at a constant speed of 450 mph, how fast is the distance between the observer and the plane changing when the distance between the plane and observer is 10 miles?

   Quantities that change with time: $x$ = Horizontal distance (See figure)
   $D$ = Distance between observer and the plane




   Rates of Change: $\frac{dD}{dt} = ?$ when $D = 10$ miles
   $\frac{dx}{dt} = 450$ mph

   Equation Relating $x$ and $D$: $x^2 + 4^2 = D^2$ or $x^2 + 16 = D^2$

Differentiation with respect to time: $\frac{d}{dt}[x^2+16] = \frac{d}{dt}[D^2]$

$$2x \cdot \frac{dx}{dt} = 2D \cdot \frac{dD}{dt}$$

$$x \cdot \frac{dx}{dt} = D \cdot \frac{dD}{dt}$$

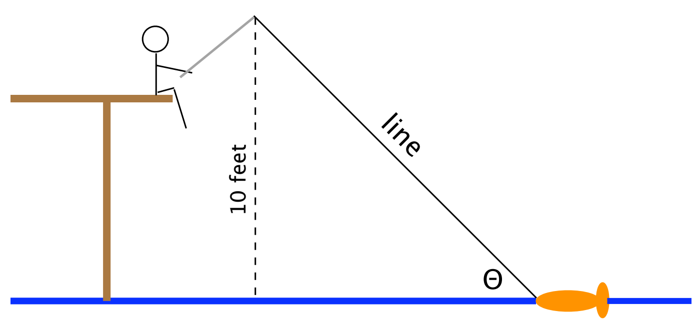
Finding $\frac{dD}{dt}$ when $D = 10$:

First we must find $x$ when $D = 10$ $\Rightarrow$ $x^2 + 16 = 10^2$ $\Rightarrow$ $x = \sqrt{84} = 2\sqrt{21}$

$$2\sqrt{21}(450) = 10 \cdot \frac{dD}{dt} \quad \Rightarrow \quad \frac{dD}{dt} = 90\sqrt{21} \text{ mph} \approx 412.4318 \text{ mph}$$

The distance between the observer and the plane is increasing by $90\sqrt{21} \approx 412.4318$ mph when the distance is 10 miles.

3. A fish is reeled in at a rate of 1 foot per second from a point 10 feet above the water (See figure). At what rate is the angle between the line and the water changing when there is a total of 25 feet of line out?




Quantities that change with time: $L$ = Length of the Line
$\theta$ = Angle between the line and the water

Rates of Change: $\frac{dL}{dt} = -1$ feet/sec (Note: $\frac{dL}{dt}$ is negative since $L$ is decreasing.)
$\frac{d\theta}{dt} = ?$ when $L = 25$

Equation Relating $L$ and $\theta$: $\sin\theta = \frac{10}{L}$ or $L \cdot \sin\theta = 10$

Differentiation with respect to time: $\frac{d}{dt}[\sin\theta] = \frac{d}{dt}[\frac{10}{L}]$

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{-10}{L^2} \cdot \frac{dL}{dt}$$

Finding $\frac{d\theta}{dt}$ when $L = 25$:

First we must find the value of $\theta$ or $\cos\theta$ when $L = 25$.

$$\Rightarrow \quad \sin\theta = \frac{10}{25} \quad \text{or} \quad \theta = \arcsin(\frac{10}{25}) \approx .4115$$

Note that if $\sin\theta = \dfrac{10}{25} = \dfrac{2}{5}$, then $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-(\frac{2}{5})^2} = \dfrac{\sqrt{21}}{5}$.

$$\Rightarrow \quad \frac{\sqrt{21}}{5} \cdot \frac{d\theta}{dt} = \frac{-10}{25^2}(-1) \qquad \Rightarrow \quad \frac{d\theta}{dt} = \frac{2}{25\sqrt{21}} \approx .01746 \text{ radians/sec}$$

The angle between the line and the water is increasing by $\dfrac{2}{25\sqrt{21}} \approx .01746$ radians/sec when there is 25 feet of line out.