Math 124
Final Exam
December 10th, 2007

Name: _____

1. Your exam contains 9 questions and 7 pages; Please make sure you have a complete exam.
2. The entire exam is worth 100 points. Point values vary and these are indicated on each problem. You have 2 hours for this exam.
3. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credit unless all work is clearly shown. If in doubt, ask for clarification. Proof by calculator does not suffice for credit. Note: To evaluate limits, proof by graph or table of values does not suffice for full credit.
4. If you need extra space, use the back of the exam and clearly indicate this.
5. You are allowed one 8.5×11 sheet of handwritten notes (both sides). Graphing and scientific calculators are permitted.
6. Leave answers in exact form or round to 4 decimal places.

Problem	Total Points	Score
1	14	
2	8	
3	7	
4	15	
5	12	
6	8	
7	14	
8	12	
9	10	
Total	100	

1. (14 pts.) Evaluate the following limits. Justify your answers. If the limit is infinite, determine if it is $+\infty$ or $-\infty$.

   (a) (5 pts.) $\lim_{x\to\infty}[2\arctan(x) - \dfrac{x^2-2x}{4x^2+5}]$

   (b) (5 pts.) $\lim_{t\to 1}\dfrac{t^3-1}{t^2-1}$

   (c) (4 pts.) $\lim_{x\to\pi^+}\dfrac{\cos x}{(x-\pi)^3}$

2. (8 pts.) Find the intervals for which $h(x) = xe^{-\frac{1}{2}x^2}$ is increasing.

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3. (7 pts.) Find the inflection point(s) of $y = \frac{3}{5}x^5 - x^4$.

4. (15 pts.) Find the following. **You do not need to simplify your answers.**

(a) (5 pts.) Find $\frac{dy}{dx}$ for $y = [\ln(\tan x) + 2x]^2$

(b) (5 pts.) $\frac{d}{dx}[\sqrt[3]{2x^4 - 5} \cdot e^{\arcsin x}] = ?$

(c) (5 pts.) Find $f'(t)$ when $f(t) = (\frac{2^t}{t} - t^2) \cdot \sec(3t)$.

5. (12 pts.) Find the slope of the tangent line of the curve $e^{xy} + 4 = 5y - 3x$ at the point on the curve at which $x = 0$.

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6. (8 pts.) At exactly 5 seconds, the velocity of a particle moving along a straight path is -10 cm/sec and the acceleration is -0.34 cm/sec^2.

   (a) (5 pts.) Approximate the velocity of the particle at 5.5 seconds using linear approximation.

   (b) (3 pts.) Is the particle speeding up or slowing down at 5 seconds?

7. (14 pts.) Find the minimum slope of the curve $y = \ln x + \frac{4}{3}x^3$. Show your work and justify that your solution is the absolute minimum.

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8. (12 pts.) For what values of x are the following functions continuous?

(a) (6 pts.) $g(x) = \begin{cases} \dfrac{-3x-3}{x^2-2x-3} & \text{if } x < 0 \\ \cos(\dfrac{x}{2}) & \text{if } x \geq 0 \end{cases}$

#8 Continued:

(b) (6 pts.) $f(x) = \ln(\sin^2 x) + \dfrac{1}{e^{x-2}}$

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9. (10 pts.) A woman that is 5 feet tall is walking away from a 14 foot tall streetlight at a rate of 4 feet per second.

   (a) (8 pts.) How fast is the **length of her shadow** changing when she is 10 feet away from the light? (Include units.)

   (b) (3 pts.) Is the length of her shadow increasing or decreasing?