Math 124
Exam 2 Solutions

1. (53 pts.) Find the following. You do not need to simplify your answers.

   (a) (9 pts.) Find $f'(x)$ if $f(x) = 2(x+3)^7 \cdot \sin(x)$.

   $$\begin{aligned} f'(x) &= 2 \cdot 7(x+3)^6 \sin(x) + 2(x+3)^7 \cos(x) \qquad \text{(Product Rule)} \\ &= 14(x+3)^6 \sin(x) + 2(x+3)^7 \cos(x) \end{aligned}$$

   (b) (10 pts.) $\frac{d}{dt}\left[\frac{e^t - 4t^3}{5 + \ln\ t}\right] = ?$

   $$\frac{d}{dt}\left[\frac{e^t - 4t^3}{5 + \ln\ t}\right] = \frac{(5 + \ln\ t)(e^t - 12t^2) - (e^t - 4t^3)(\frac{1}{t})}{(5 + \ln\ t)^2} \qquad \text{(Quotient Rule)}$$

   (c) (12 pts.) Find $\frac{dy}{dx}$ if $y = (\arctan\ x)^{\sqrt{x}}$.

   Logarithmic Differentiation:
   $$\begin{aligned} \ln\ y &= \ln(\arctan\ x)^{\sqrt{x}} \\ \ln\ y &= \sqrt{x} \cdot \ln(\arctan\ x) \\ \Rightarrow \quad \frac{d}{dx}[\ln\ y] &= \frac{d}{dx}[\sqrt{x} \cdot \ln(\arctan\ x)] \\ \frac{1}{y}\frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \cdot \ln(\arctan\ x) + \sqrt{x} \cdot \frac{1}{\arctan\ x} \cdot \frac{1}{1+x^2} \end{aligned}$$

   $$\Rightarrow \quad \frac{dy}{dx} = (\arctan\ x)^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}} \cdot \ln(\arctan\ x) + \sqrt{x} \cdot \frac{1}{\arctan\ x} \cdot \frac{1}{1+x^2}\right]$$

   (d) (12 pts.) Find $g''(t)$ if $g(t) = \cos(\ln\ t) + 3t$.

   $$g'(t) = -\sin(\ln\ t) \cdot \frac{1}{t} + 3 \qquad \text{(Chain Rule))}$$

   $$\begin{aligned} g''(t) &= -\ \cos(\ln\ t) \cdot \frac{1}{t} \cdot \frac{1}{t} - \sin(\ln\ t) \cdot -\frac{1}{t^2} \\ &= -\cos(\ln\ t) \cdot \frac{1}{t^2} + \sin(\ln\ t) \cdot \frac{1}{t^2} \qquad \text{(Chain and Product Rule)} \end{aligned}$$

   (e) (10 pts.) $\frac{d}{d\theta}[e^{\sec^2\theta}] = ?$

   $$\frac{d}{d\theta}[e^{\sec^2\theta}] = e^{\sec^2\theta} \cdot 2\sec\ \theta \cdot \sec\ \theta \cdot \tan\ \theta \qquad \text{(Chain Rule)}$$

2. (10 pts.) Evaluate the following limit. $\lim_{x \to 0} \frac{\cot\ x}{3\csc\ x}$

   $$\lim_{x \to 0} \frac{\cot\ x}{3\csc\ x} = \lim_{x \to 0} \frac{\frac{\cos\ x}{\sin\ x}}{\frac{3}{\sin\ x}} = \lim_{x \to 0} \frac{\cos\ x}{\sin\ x} \cdot \frac{\sin\ x}{3} = \lim_{x \to 0} \frac{\cos\ x}{3} = \frac{1}{3}$$

3. (19 pts.) Find an equation of the tangent line to the curve $\tan(xy) = x^3 + 2y - 4$ at the point (0,2).

Implicit Differentiation: $\frac{d}{dx}[\tan(xy)] = \frac{d}{dx}[x^3 + 2y - 4]$

$$\sec^2(xy) \cdot (x\frac{dy}{dx} + y) = 3x^2 + 2\frac{dy}{dx}$$

Plugging in $x = 0$ and $y = 2$: $\sec^2(0 \cdot 2) \cdot (0\frac{dy}{dx} + 2) = 3(0)^2 + 2\frac{dy}{dx}$

$$2 = 2\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = 1$$

So, the equation of the tangent line is $y = x + 2$.

4. (18 pts.) The **velocity** of a flying monkey is given by $v(t) = (6 - t)e^{0.5t}$ in meters/second at t seconds.

   (a) (9 pts.) When is the monkey at rest? When is the monkey moving forward?

   The monkey is at rest when the velocity is equal to 0.

   $(6 - t)e^{0.5t} = 0$ when $t = 6$ (since $e^{0.5t} > 0$ for all t)

   So, the monkey is at rest at 6 seconds.

   The monkey is moving forward when the velocity is positive.

   $(6 - t)e^{0.5t} > 0$ when $6 - t > 0$ (since $e^{0.5t} > 0$ for all t)

   $\Rightarrow \quad 6 > t$

   So, the monkey is moving forward between 0 and 6 seconds.

   (b) (9 pts.) Find the acceleration $a(t)$ of the monkey at time t.

   Acceleration: $a(t) = -e^{0.5t} + (6 - t)e^{0.5t} \cdot 0.5$ (Product and Chain Rule)