Math 112 Worksheet #8 Solutions

1. You win the state lottery and have the choice between a lump sum of \$7.5 million, paid immediately, or \$10 million paid continuously over 10 years. You can invest at an interest rate of 7% compounded continuously. Which option is better? What if the interest rate is 4%?

<u>7%:</u> The present value of the annuity will be given by

$$\int_0^{10} S(t)e^{-.07t}\,dt = \int_0^{10} 1000000e^{-.07t}\,dt = \frac{1000000}{-.07}e^{-.07t}\Big|_0^{10}$$

$$= \$7{,}191{,}638.52$$

Since the present value of the annuity is less than the lump sum payment of \$7.5 million, the lump sum option is better.

<u>4%:</u> The present value of the annuity will be given by

$$\int_0^{10} S(t)e^{-.04t}\,dt = \int_0^{10} 1000000e^{-.04t}\,dt = \frac{1000000}{-.04}e^{-.04t}\Big|_0^{10}$$

$$= \$8{,}241{,}998.85$$

Since the present value of the annuity is more than the lump sum payment of \$7.5 million, the annuity option is better.

2. Suppose the number of pounds of apples purchased by the average household monthly is a function of the price of apples a per pound and the price of pears p per pound. (Apples purchased $= f(a,p)$) Values of f are given in the table below for particular prices of apples and pears.

Price of Pears p

	\$1.00	\$2.00	\$3.00
\$.50	14	17	20
\$1.00	11	13	15
\$1.50	9	11	12

(a) Find $f(1,2)$ and interpret it in terms of the number of pounds of apples purchased.

Looking at the table ($a=1$ and $p=2$), we have that $f(1,2)=13$. This means that if the price of apples is \$1 per pound and the price of pears is \$2 per pound, then the average household will buy 13 pounds of apples monthly.

(b) Is f an increasing or decreasing (or neither) function of a? Is f an increasing or decreasing (or neither) function of p?

Consider how f changes as we let a increase, but hold p constant (look along the columns of the table). We can see that f is a decreasing function of a. (As price of apples increases, the quantity of apples purchased decreases.) Similarly, looking at how f changes as p increases (look along the rows), we see that f is an increasing function of p. (As price of pears increases, the quantity of apples purchased increases.)

(c) Estimate $f_a(1,2)$ and $f_p(1,2)$.

- The partial derivative of f with respect to a is the rate of change of f as a increases holding p constant. To estimate $f_a(1,2)$, consider the average rates of change between the points $f(1,2)$ and $f(1.5,2)$ and between $f(1,2)$ and $f(.5,2)$.

  Average rate of change from $a = .5$ to $a = 1$: $\frac{f(1,2)-f(.5,2)}{1-.5} = \frac{13-17}{.5} = -8$

  Average rate of change from $a = 1$ to $a = 1.5$: $\frac{f(1.5,2)-f(1,2)}{1.5-1} = \frac{11-13}{.5} = -4$

  Averaging the two rates: $f_a(1,2) \approx \frac{1}{2}(-8 + -4) = -6$

- The partial derivative of f with respect to p is the rate of change of f as p increases holding a constant. To estimate $f_p(1,2)$, consider the average rates of change between the points $f(1,2)$ and $f(1,1)$ and between $f(1,2)$ and $f(1,3)$.

  Average rate of change from $p = 1$ to $p = 2$: $\frac{f(1,2)-f(1,1)}{2-1} = \frac{13-11}{1} = 2$

  Average rate of change from $p = 2$ to $p = 3$: $\frac{f(1,3)-f(1,2)}{3-2} = \frac{15-13}{1} = 2$

  Averaging the two rates: $f_p(1,2) \approx 2$

(d) Estimate $f(1.25, 1.5)$ given your answers to part (a) and (c).

Given that $f_a(1,2) \approx -6$ and $f_p(1,2) = 2$, we have that

$$f(1.25, 1.5) \approx 13 + .25(-6) - .5(2) = 10.5 \text{ pounds of apples}$$

3. If $f(x,y) = 2x^3 + 3xy^2 - 5y$, then which is greater, $f_x(1,2)$ or $f_y(1,2)$?

Holding y constant: $f_x = 6x^2 + 3y^2$

Holding x constant: $f_y = 6xy - 5$

So, $f_x(1,2) = 6(1)^2 + 3(2)^2 = 18$ and $f_y(1,2) = 6(1)(2) - 5 = 7$. $\Rightarrow f_x(1,2)$ is greater.