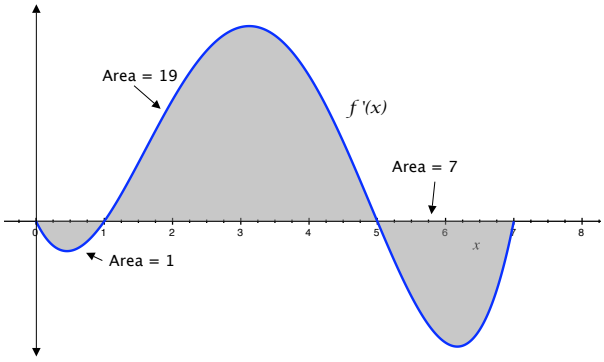
Math 112 Worksheet #7 Solutions

1. Sketch a graph of f given that $f(5) = 2$ and the graph of $f'(x)$ below. What are the global maximum and minimum values of f?

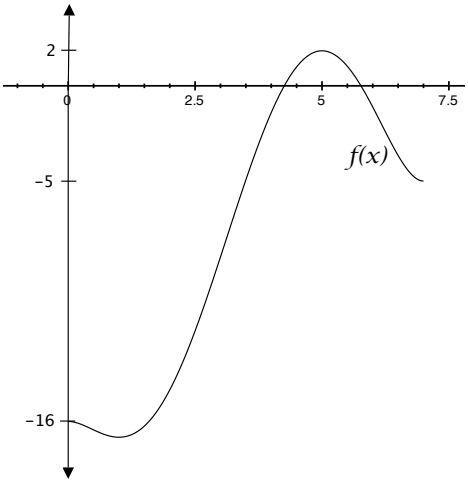


Finding Values for $f(0)$, $f(1)$, and $f(7)$:

- $f(7) = f(5) + \int_5^7 f'(x)\,dx = 2 - 7 = -5$
- $f(1) = f(5) - \int_1^5 f'(x)\,dx = 2 - 19 = -17$ since $f(5) = f(1) + \int_1^5 f'(x)\,dx$
- $f(0) = f(1) - \int_0^1 f'(x)\,dx = -17 - (-1) = -16$ since $f(1) = f(0) + \int_0^1 f'(x)\,dx$

The critical points and endpoints of $f(x)$ are $x = 0$, $x = 1$, $x = 5$, and $x = 7$, so these are the only possible points at which we can have a global maximum or minimum.
Looking at the values of f at these points, we can see that there is a global max at $x = 5$ ($f(5) = 2$) and a global min at $x = 1$ ($f(1) = -17$).

Here's a sketch of the graph of $f(x)$.



2. The temperature of a baked potato in Fahrenheit t minutes after being taken out of the oven is given by $P(t) = 65 + 285e^{-.05t}$.

   (a) What is the temperature of the potato when it is taken out of the oven?

   The temperature when it is taken out of the oven is $P(0) = 65 + 285e^0 = 350$ degrees Fahrenheit.

   (b) What is the temperature of the potato one hour after it is taken out of the oven?

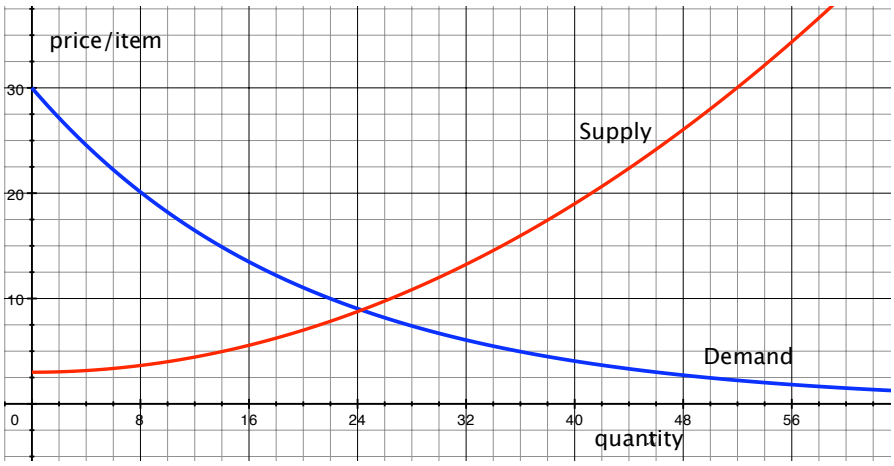
   The temperature after one hour is $P(60) = 65 + 285e^{-.05(60)} = 79.189$ degrees Fahrenheit.

   (c) What is the average value of the temperature from time over the first hour?

   The average value of the temperature over the first hour is given by

$$\frac{1}{60-0}\int_0^{60} 65+285e^{-.05t}\,dt = \frac{1}{60}(65t+\frac{285}{-.05}e^{-.05t})\,\Big|_0^{60} = \frac{1}{60}(3616.21-(-5700)) = 155.27 \text{ degrees}$$

3. Graphs of supply and demand curves for selling gigantic erasers are given below.



   (a) Estimate the consumer and producer surplus if the market is in equilibrium.

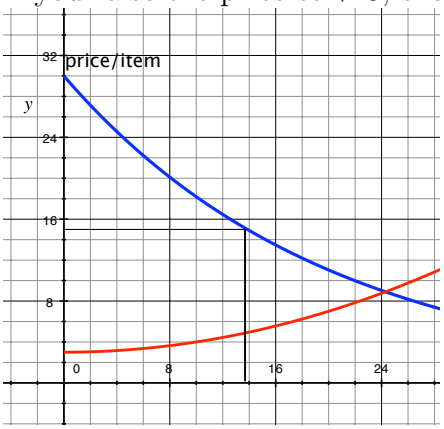
   The equilibrium quantity is approximately $q = 25$ erasers and the equilibrium price is approximately $p = 9$ dollars.

   Looking at the area above \$9 and below the demand curve gives us the consumer surplus. The area is approximately $200 \Rightarrow$ CS $\approx$ \$200.

The area below \$9 and above the supply curve is equivalent to the producer surplus. The area is approximately 100 $\Rightarrow$ PS $\approx$ \$100.

(b) Estimate the consumer and producer surplus if the price is kept artificially high at \$15.

If you raise the price to \$15, the quantity sold is approximately 13.75 erasers.



The consumer surplus is the area above \$15 and below the demand curve. The area is approximately 90 $\Rightarrow$ CS $\approx$ \$90.

The producer surplus is the area below \$15 and above the supply curve. The area is approximately 155 $\Rightarrow$ PS $\approx$ \$155.