Math 112 Worksheet #6 Solutions

1. (a) Estimate $\int_0^2 x^2\,dx$ using 4 rectangles. Is your estimate an overestimate, an underestimate, or is it difficult to distinguish whether it is either?

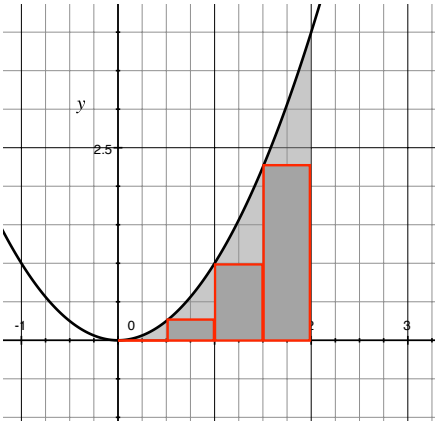
4 rectangles $\Rightarrow$ width Δx of each rectangle is .5.

Using a left-end sum of rectangles, we have the following sum:
Area of left-end rectangles $= \Delta x \cdot f(0) + \Delta x \cdot f(.5) + \Delta x \cdot f(1) + \Delta x \cdot f(1.5)$
$= .5(0) + .5(.25) + .5(1) + .5(2.25) = 1.75$

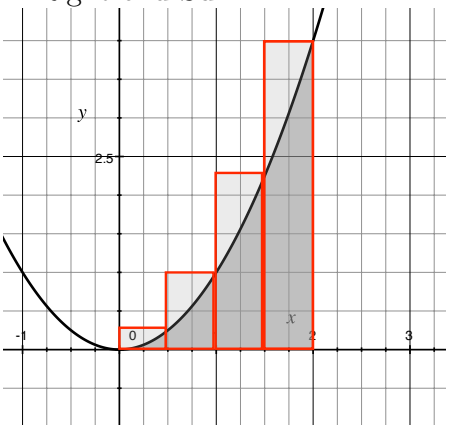
Using a right-end sum of rectangles, we have the following sum:
Area of right-end rectangles $= \Delta x \cdot f(.5) + \Delta x \cdot f(1) + \Delta x \cdot f(1.5) + \Delta x \cdot f(2)$
$= .5(.25) + .5(1) + .5(2.25) + .5(4) = 3.75$

Averaging the two values, we have $\frac{1}{2}(1.75 + 3.75) = 2.75$.

Left-end Sum



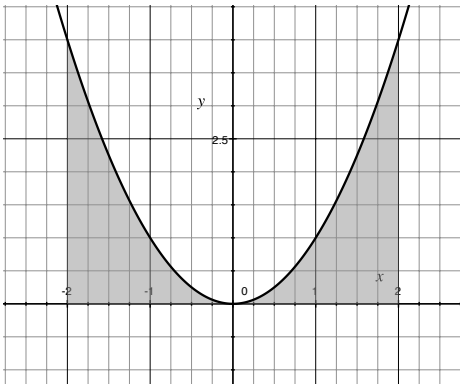
Right-end Sum



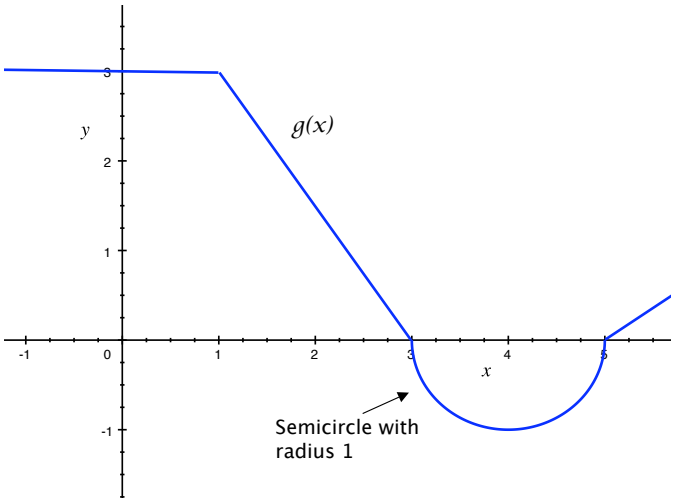
If you choose to estimate using the left-end sum, then the estimate will be an underestimate.
If you choose to estimate using the right-end sum, then the estimate will be an overestimate.
If you choose to estimate using the average of the left and right-end sum, it is hard to determine whether the estimate is an overestimate or underestimate.

(b) Given your answer to (a), estimate $\int_{-2}^2 x^2\,dx$.

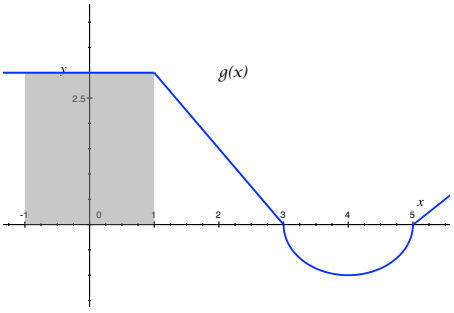
Since x^2 is symmetric about the y-axis, to estimate $\int_{-2}^2 x^2\,dx$, we can double our estimate from part (a). The shaded area in the figure below represents $\int_{-2}^2 x^2\,dx$.



2. The graph of g is given below. Find the following values.

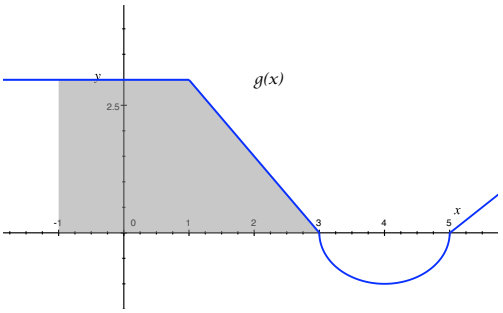


(a) $\int_{-1}^{1} g(x)\,dx$



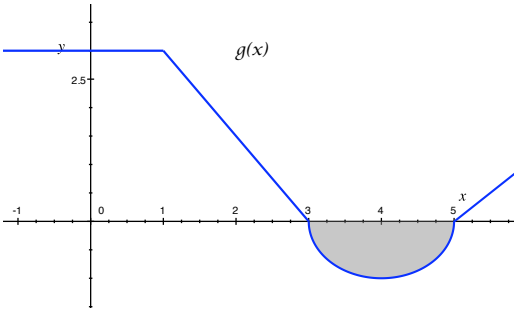
$$\int_{-1}^{1} g(x)\,dx = 2 \cdot 3 = 6$$

(b) $\int_{-1}^{3} g(x)\,dx$



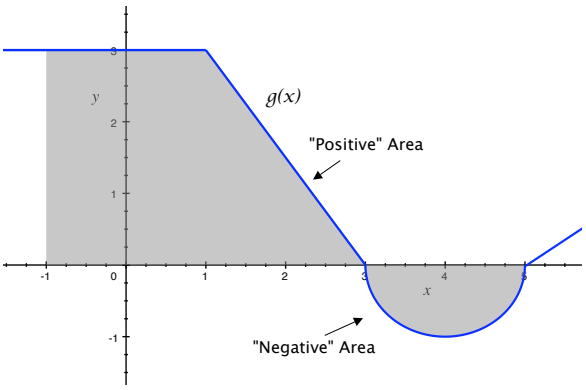
$$\int_{-1}^{3} g(x)\,dx = 6 + \frac{1}{2}(2)(3) = 9$$

(c) $\int_{3}^{5} g(x)\,dx$



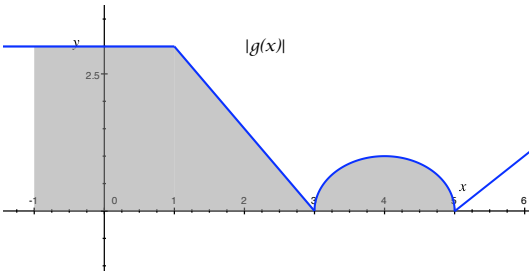
$$\int_{3}^{5} g(x)\,dx = -\frac{\pi}{2} \text{ (Semicircle with } r = 1)$$

(d) $\int_{-1}^{5} g(x)\,dx$



$$\int_{-1}^{5} g(x)\,dx = 6 + 3 - \frac{\pi}{2} \approx 7.43$$

(e) $\int_{-1}^{5} |g(x)|\,dx$

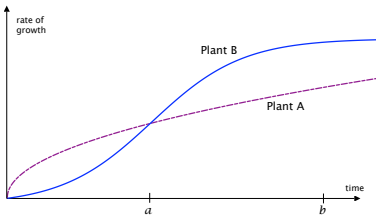


$$\int_{-1}^{5} |g(x)|\,dx = 6 + 3 + \frac{\pi}{2} \approx 10.57$$

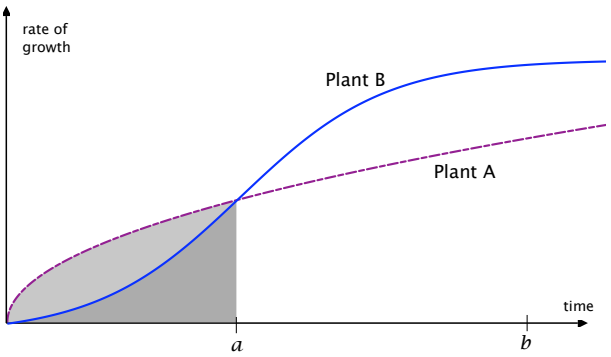
3. For velocity $v(t)$ in ft/sec and t in seconds, what does $\int_0^{20} v(t)\,dt$ represent and give units.

$\int_0^{20} v(t)\,dt$ = Total distance traveled in miles between 0 and 20 seconds.

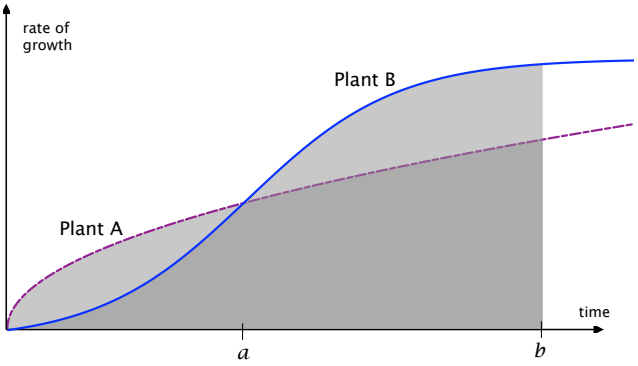
4. The graph below shows the rates of growth in inches per year of two plants for a given year. Assuming both plants began as seeds at time 0, which plant is taller at time a? Which plant is taller at time b?



Time a: Looking at the areas under the rate of growth curves from time 0 to time a, we can see that the area under the curve corresponding to plant A is larger than the area under the curve for B. Thus, the total growth of plant A is larger than the total growth for plant B from times 0 to $a \Rightarrow$ Plant A is taller.



Time b: Looking at the areas under the rate of growth curves from time 0 to time b, we can see that the area under the curve corresponding to plant B is larger than the area under the curve for A. Thus, the total growth of plant B is larger than the total growth for plant A from times 0 to $b \Rightarrow$ Plant B is taller.


rate of
growth
Plant B
Plant A
time
a
b