Math 112 Worksheet #5 Solutions

1. Antiderivatives:

   (a) $\int 2z^3 - z^{1/3} + 5\,dz = ?$

   $$\int 2z^3 - z^{1/3} + 5\,dz = 2(\tfrac{1}{4})z^4 - z^{4/3}(\tfrac{3}{4}) + 5z + C$$
   $$= \tfrac{1}{2}z^4 - \tfrac{3}{4}z^{4/3} + 5z + C$$

   (b) $\int \frac{3}{t} - e^{2t}\ dt = ?$

   $$\int \frac{3}{t} - e^{2t}\ dt = 3\ln|t| - \tfrac{1}{2}e^{2t} + C$$

   (c) $\int xe^{5x^2}\ dx = ?$

   Let $u = 5x^2$. Then $du = 10x\ dx$ OR $\frac{1}{10}\ du = x\ dx$.

   Then $\int xe^{5x^2}\ dx = \int e^u \cdot \dfrac{1}{10}\,du = \frac{1}{10}e^u + C = \frac{1}{10}e^{5x^2} + C$

   (d) $\int \sqrt{5s+2}\,ds = ?$

   Let $u = 5s + 2$. Then $du = 5\ ds$ OR $\frac{1}{5}\ du = ds$.

   Then $\int \sqrt{5s+2}\,ds = \int \sqrt{u} \cdot \dfrac{1}{5}\,du = \frac{1}{5}(\frac{2}{3})u^{3/2} + C = \frac{2}{15}(5s+2)^{3/2} + C$

   (e) $\int 8t^3(t^4+5)^6\,dt = ?$

   Let $u = t^4 + 5$. Then $du = 4t^3\ dt$.

   Then $\int 8t^3(t^4+5)^6\,dt = \int 2u^6\,du = 2(\frac{1}{7})u^7 + C = \frac{2}{7}(t^4+5)^7 + C$

   (f) $\int \dfrac{\ln\ q}{q}\ dq = ?$

   Let $u = \ln\ q$. Then $du = \dfrac{1}{q}\ dq$.

   Then $\int \dfrac{\ln\ q}{q}\ dq = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln\ q)^2 + C$

2. The table below gives the velocity v of a squirrel (in meters/min). Assuming the squirrel's velocity is always increasing, find upper and lower estimates of the total distance the squirrel has traveled between $t = 0$ and $t = 4$.

Time (min)	0	1	2	3	4
Velocity (m/min)	0	1.25	4	10.25	24.5

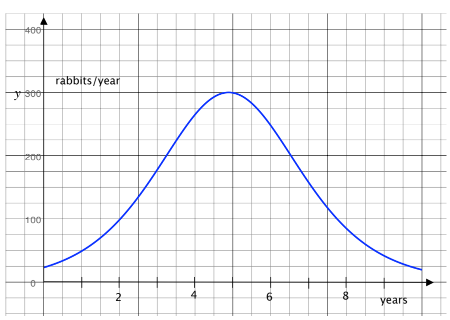
Lower Estimate:
0 to 1 sec: The velocity ≥ 0 m/min. $\rightarrow$ Distance ≥ 0 meters.
1 to 2 sec: The velocity ≥ 1.25 m/min. $\rightarrow$ Distance ≥ 1.25 meters.
2 to 3 sec: The velocity ≥ 4 m/min. $\rightarrow$ Distance ≥ 4 meters.
3 to 4 sec: The velocity ≥ 10.25 m/min. $\rightarrow$ Distance ≥ 10.25 meters.

So, from 0 to 4 seconds, the squirrel travels at **least** $0+1.25+4+10.25 = 15.5$ meters.

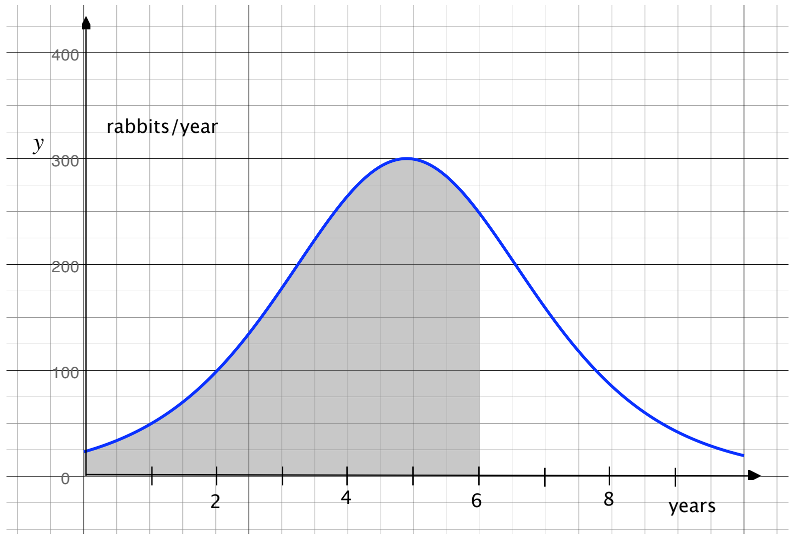
Upper Estimate:
0 to 1 sec: The velocity ≥ 1.25 m/min. $\rightarrow$ Distance ≥ 1.25 meters.
1 to 2 sec: The velocity ≥ 4 m/min. $\rightarrow$ Distance ≥ 4 meters.
2 to 3 sec: The velocity ≥ 10.25 m/min. $\rightarrow$ Distance ≥ 10.25 meters.
3 to 4 sec: The velocity ≥ 24.5 m/min. $\rightarrow$ Distance ≥ 24.5 meters.

So, from 0 to 4 seconds, the squirrels travels at **most** $1.25+4+10.25+24.5 = 40$ meters.

3. The graph below gives the rate of change of a population of rabbits on an island for a given year t. Estimate the total change in population from $t = 0$ and $t = 6$.



The total change in population is represented by the shaded area below.



You can estimate this in various ways. One way would be to count the number of rectangles (from the grid) in the shaded area. There are approximately 80 rectangles and each rectangles represents a change of .5 years and 25 rabbits/year. So, each rectangle represents 12.5 rabbits. Thus, there was a change of about 80(12.5)=1000 rabbits.

Another way is to use rectangles such as the those in the figure below. In this example, I have drawn right-end rectangles.