

Math 112 Worksheet #4 Solutions

1. Find the local and global maxima and minima for the function $f(x) = 50(e^{-.5x} - e^{-x})$ on the interval $1 \leq x \leq 10$. Sketch a graph of f .

The derivative of f is $f'(x) = 50(-.5e^{-.5x} - (-1)e^{-x}) = 50(-.5e^{-.5x} + e^{-x})$. This is defined for all x -values.

$$\begin{aligned} \text{Critical Point(s): } 50(-.5e^{-.5x} + e^{-x}) = 0 &\Rightarrow -.5e^{-.5x} + e^{-x} = 0 \\ &\Rightarrow e^{-x} = .5e^{-.5x} \\ &\Rightarrow \frac{e^{-x}}{e^{-.5x}} = .5 \\ &\Rightarrow e^{-.5x} = .5 \\ &\Rightarrow \ln(e^{-.5x}) = \ln(.5) \\ &\Rightarrow -.5x = \ln(.5) \\ &\Rightarrow x = -2\ln(.5) \end{aligned}$$

Evaluating f at the critical points and endpoints:

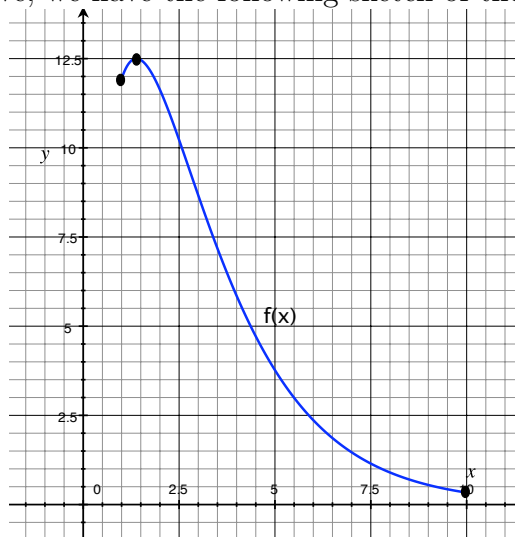
$$f(1) = 50(e^{-.5} - e^{-1}) \approx 11.9326 \qquad f(10) = 50(e^{-5} - e^{-10}) \approx 0.3346$$

$$f(-2\ln(.5)) = 50(e^{\ln(.5)} - e^{2\ln(.5)}) = 50(.5 - .25) = 12.5$$

The global maximum of f occurs at $x = -2\ln(.5) = \ln(4) \approx 1.386$ and the global minimum occurs at $x = 10$.

Given that there is only one critical point of f , we must have that f is increasing between $x = 1$ and $x \approx 1.386$. So, we must have a local minimum at $x = 1$.

Using the points found above, we have the following sketch of the f :



2. Suppose you sell widgets for \$300 each. The marginal cost for producing widgets is $MC(q) = q^2 - 70q + 1300$. Should you increase production if you have already

produced 30 items? 70 items? How many widgets should you sell to maximize profit?

Since $MC(30) = 30^2 - 70(30) + 1300 = 100 < 300 = MR(30)$, we should increase production to 31 items.

Since $MC(70) = 70^2 - 70(70) + 1300 = 1300 > 300 = MR(70)$, we should not increase production to 71 items.

Since $MR = MC$ when profit is maximized, we can find possible quantities for which profit is maximized by setting $MR(q)$ equal to $MC(q)$.

$$\Rightarrow q^2 - 70q + 1300 = 300$$

$$\Rightarrow q^2 - 70q + 1000 = 0$$

$$\Rightarrow (q - 50)(q - 20) = 0$$

So, $MR = MC$ when $q = 20$ and $q = 50$.

To check if $q = 20$ is a maximum or minimum of profit, consider the marginal cost at $q = 19$ and $q = 21$.

$$MC(19) = 331 > MR(19)$$

$$MC(21) = 271 < MR(21)$$

Since we switch from $MC > MR$ to $MR > MC$ at $q = 20$, profit is minimized at $q = 20$.

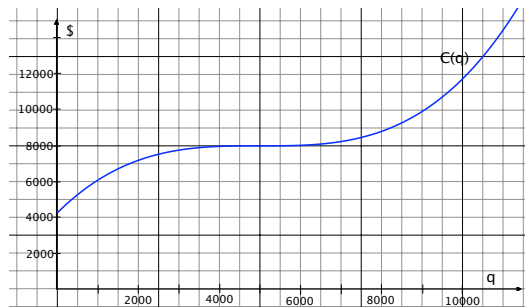
To check if $q = 50$ is a maximum or minimum of profit, consider the marginal cost at $q = 49$ and $q = 51$.

$$MC(49) = 271 < MR(49)$$

$$MC(51) = 331 > MR(51)$$

Since we switch from $MC < MR$ to $MR < MC$ at $q = 50$, profit is maximized at $q = 50$.

3. The total cost of producing cheese graters is shown in the graph below.

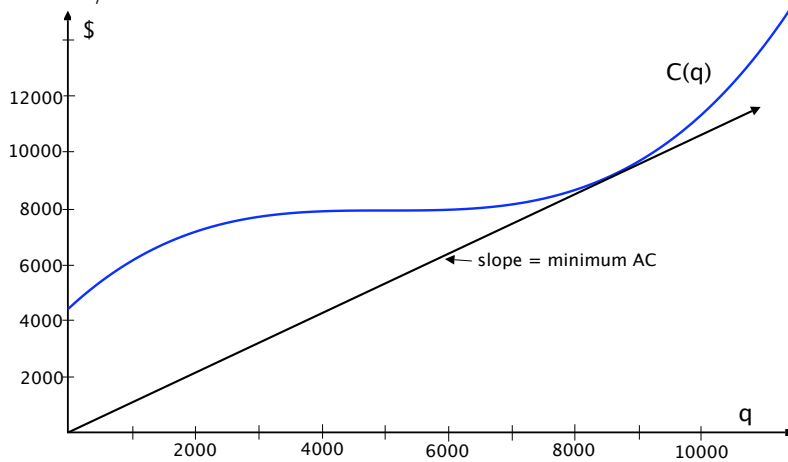


(a) Estimate the average cost of producing 5000 graters.

$$AC(5000) = \frac{C(5000)}{5000} \approx \frac{8000}{5000} = 1.6 \text{ dollars/unit}$$

(b) Estimate the minimum average cost (i.e. the breakeven price).

Graphing the line that gives us the minimum average cost and picking the point (8700, 9200) on the line we have that the minimum average cost $\approx \frac{9200}{8700} = 1.06$ dollars/unit.



- (c) The following table shows the total sales (in thousands) of the cheese graters t months after the product is released. Approximately when is the point of diminishing returns reached? Estimate the total possible sales.

Month	2	4	6	8	10	12	14
Sales	16.8	41.8	93.1	169.6	243.1	289.2	310.8

Consider the differences in sales between the 2 month periods.

Time Interval	2-4	4-6	6-8	8-10	10-12	12-14
Difference in Sales	25	51.3	76.5	73.5	46.1	21.6

This table shows that the rate of change of total sales increases until $t \approx 8$ months and the rate of change of total sales decreases after $t \approx 8$. So, we have expect the point of diminishing returns of this function (the inflection point) to be around $t = 8$.

At the point of diminishing returns, we have that the sales should be half of the limiting value (maximum value of total sales). Sales at $t = 8$ are 169,600. So, we can expect the total possible sales to be approximately $2(169,600)=339,200$ units.