Math 112 Worksheet #3 Solutions

1. Differentiate the following:

   (a) $f(x) = 2x^4 - 3x^2 + \pi$

   $f'(x) = 2(4x^3) - 3(2x) + 0 = 8x^3 - 6x$

   (b) $g(t) = \sqrt[3]{t} + \dfrac{3}{t^3}$

   Note that $\sqrt[3]{t} = t^{1/3}$ and $\dfrac{3}{t^3} = 3t^{-3}$.

   $g'(t) = \frac{1}{3}t^{-2/3} + 3(-3t^{-4}) = \frac{1}{3}t^{-2/3} - 9t^{-4} = \dfrac{1}{3t^{2/3}} - \dfrac{9}{t^4}$

   (c) $h(z) = 12e^{.5z} - \ln 9z$

   Using the chain rule for each term, we have $h'(z) = 12(.5e^{.5z}) - \dfrac{1}{z}$
   $= 6e^{.5z} - \dfrac{1}{z}$

   (d) $2(p^2+3)^4$

   Using the chain rule, we have $\frac{\mathrm{d}}{\mathrm{d}p}(2(p^2+3)^4) = 2(4(p^2+3)^3) \cdot \frac{\mathrm{d}}{\mathrm{d}p}(p^2+3)$
   $= 8(p^2+3)^3 \cdot (2p+0)$
   $= 16p(p^2+3)^3$

   (e) $3\sqrt{s^2-1} + \ln(s+1)$

   Using the chain rule for each term, we have

   $\frac{\mathrm{d}}{\mathrm{d}s}(3\sqrt{s^2-1} + \ln(s+1)) = 3(\frac{1}{2}(s^2-1)^{-1/2} \cdot \frac{\mathrm{d}}{\mathrm{d}s}(s^2-1) + \dfrac{1}{s+1} \cdot \frac{\mathrm{d}}{\mathrm{d}s}(s+1)$
   $= \frac{3}{2}(s^2-1)^{-1/2} \cdot (2s) + \dfrac{1}{s+1} \cdot 1$
   $= \dfrac{3s}{\sqrt{s^2-1}} + \dfrac{1}{s+1}$

2. Find the equation of the line tangent to the graph of $f(t) = 4^t - 3$ at $t = 1$.

   The derivative of f is $f'(t) = (\ln(4)) \cdot 4^t + 0 = (\ln(4)) \cdot 4^t$.

   So, the slope of f at $x = 1$ is $f'(1) = (\ln(4)) \cdot 4 = 4\ln(4) \approx 5.54518$.

   The tangent line goes through the point $(1, f(1)) = (1, 4-3) = (1, 1)$.

Using the point-slope form of the line, we have that the tangent line of f at $x = 1$ is $y - 1 = 4\ln(4)(x - 1)$

3. Suppose the cost and revenue functions for selling gizmos are $C(q) = 10000 + 5q^{1.3}$ and $R(q) = 1200\sqrt{q}$. Assuming you have sold 200 gizmos, should you sell the 201^{st} gizmo? Assuming you have sold 400 gizmos, should you sell the 401^{st} gizmo?

   Note that the marginal cost is given by $MC(q) = C'(q) = 0 + 5(1.3q^{.3}) = 6.5q^{.3}$ and the marginal revenue is given by $MR(q) = 1200(\frac{1}{2}q^{-1/2}) = 600q^{-1/2} = \dfrac{600}{\sqrt{q}}$.

   The cost of the 201^{st} gizmo is $MC(200) = 6.5(200)^{.3} \approx 31.86$ dollars.
   The revenue from the 201^{st} gizmo is $MR(200) = \dfrac{600}{\sqrt{200}} \approx 42.43$ dollars.
   Since $MC(200) < MR(200)$, you should sell the 201^{st} gizmo.

   The cost of the 401^{st} gizmo is $MC(400) = 6.5(400)^{.3} \approx 39.22$ dollars.
   The revenue from the 401^{st} gizmo is $MR(400) = \dfrac{600}{\sqrt{400}} = 30$ dollars.
   Since $MC(400) > MR(400)$, you should <u>not</u> sell the 401^{st} gizmo.