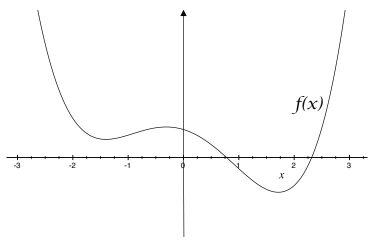
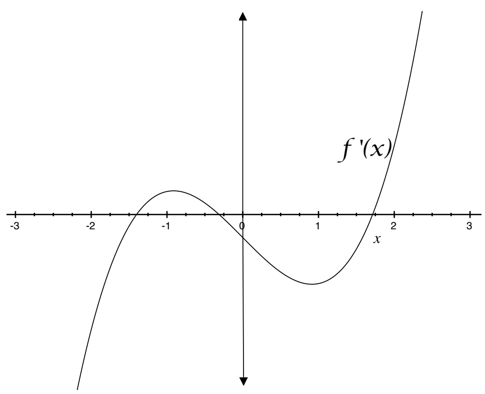
Math 112 Worksheet #2 Solutions

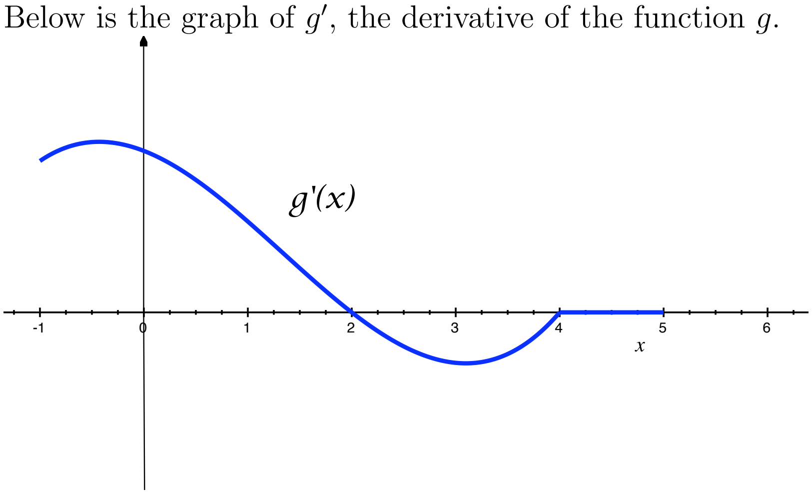
1. Sketch the graph of $f'(x)$.



Looking at the graph of f, we can estimate f has horizontal tangents around $x = -1.4$, $x = -.3$, and $x = 1.7$. So, for these x-values, we have that f' is zero. For $x < -1.4$, f is decreasing, so $f' < 0$. For $-1.4 < x < -.3$, f is increasing, so $f' > 0$ on that interval. For $-.3 < x < 1.7$, f is decreasing, so $f' < 0$. For $x > 1.7$, f is increasing, so $f' > 0$. When sketching, it is also important to note how quickly f is increasing or decreasing. For example, for $x > 1.7$, f increases more rapidly as x gets larger (the slope becomes steeper), so f' becomes more positive. A sketch of $f'(x)$ is given below.



2. Below is the graph of g', the derivative of the function g.



(a) On what interval(s) is the function g increasing, decreasing, or constant?

The function g is increasing on intervals for which g' is positive. So g is increasing for $-1 < x < 2$.
The function g is decreasing on intervals for which g' is negative. So g is decreasing for $2 < x < 4$.
The function g is constant on intervals for which g' is zero. So g is constant for $4 < x < 5$.

(b) On what approximate interval(s) is g'', the second derivative of g, positive or negative?

Note that g'' is the derivative of g'. So, $g''(x) = $ slope of g' at x.

The second derivative g'' is positive at x-values for which g' has positive slope. Estimating from the graph, g' has positive slope on the intervals $-1 < x < -.4$ and $3.1 < x < 4$. So, g'' is positive on the intervals $-1 < x < -.4$ and $3.1 < x < 4$.

The second derivative g'' is negative at x-values for which g' has negative slope. Estimating from the graph, g' has negative slope on the interval $-.4 < x < 3.1$. So, g'' is negative on the interval $-.4 < x < 3.1$.

3. The cost C (in thousands of dollars) of growing x acres of corn is given by the function $C = h(x)$.

(a) Suppose $h(100) = 37.6$ and $h'(100) = .35$. Interpret each of these statements in terms of the cost of growing corn. What are the units of $h'(100)$?

The statement $h(100) = 37.6$ means that growing 100 acres of corn will cost \$37600. The statement $h'(100) = .35$ means that the rate of change of cost when growing 100 acres of corn is \$350 per acre. So, the 101st acre will cost approximately \$350. The units of $h'(100)$ is thousands of dollars per acre. Note that another way to express the derivative is $h'(100) = \frac{dC}{dx}\Big|_{x=100}$.

(b) Estimate the cost of growing 97 acres, 101 acres, and 102 acres of corn.

We can us a local linear approximation to estimate the cost for 97, 101, and 102 acres given the information for 100 acres. Assuming h' does not change rapidly, we have that with each acre that we add, we increase costs by \$.35 thousands of dollars.
Since 97 acres is 3 less acres than 100 acres, $h(97) \approx 37.6 - 3(.35) = 36.55$.
Similarly, $h(101) \approx 37.6 + .35 = 37.95$ and $h(102) \approx 37.6 + 2(.35) = 38.3$.

So, the cost for 97 acres is \$36550, the cost for 101 acres is \$37950, and the cost for 102 acres is \$38300.