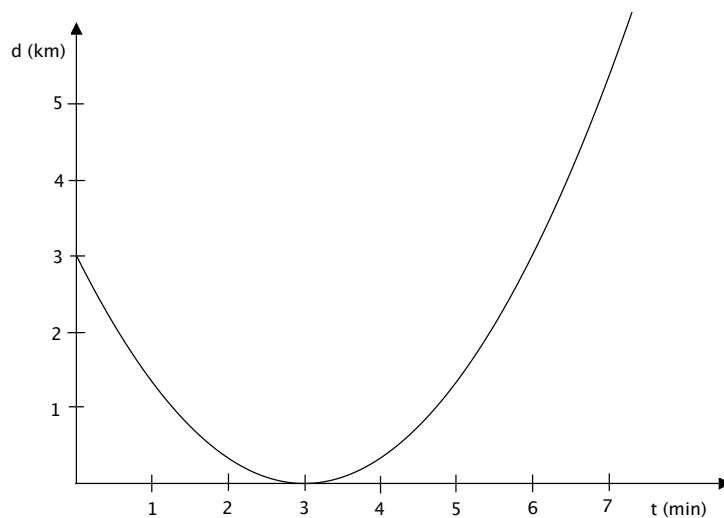


Math 112 Worksheet #1 Solutions

1. The figure below shows the distance d from home in kilometers as a function of time t in minutes. ($d = f(t)$)



- (a) Estimate the instantaneous velocity at $t = 5$ using the graph. Include units.

Graphically, the instantaneous velocity at $t = 5$ is the slope of the tangent line of f at $t = 5$. Sketching this line, you should find that the slope of the line is approximately $\frac{4}{3}$.

- (b) List the following from least to greatest:

The instantaneous
velocity at $t = 1$

The instantaneous
velocity at $t = 4$

The instantaneous
velocity at $t = 6$

Looking at the graph, the instantaneous velocity at $t = 1$ is negative since the tangent line of f at $t = 1$ has negative slope. Both the instantaneous velocity at $t = 5$ and $t = 6$ are positive since the tangent lines of f at $t = 5$ and $t = 6$ have positive slope. The tangent line of f at $t = 6$ is steeper than the tangent line of f at $t = 5$. Thus

$$(\text{Instant. vel. at } t = 1) < (\text{Instant. vel. at } t = 5) < (\text{Instant. vel. at } t = 6)$$

- (c) The distance function can also be described by the equation $f(t) = \frac{1}{3}(t-3)^2$. Use this equation to find an estimate of $f'(5)$. Include units.

To estimate $f'(5)$, consider the average velocity from $t = 5$ to $t = 5.1$. (You can make other choices for intervals that contain $t = 5$.)

$$\text{Avg. vel. from } t = 5 \text{ to } t = 5.1 = \frac{f(5.1) - f(5)}{5.1 - 5} = \frac{1.47 - \frac{4}{3}}{.1} \approx 1.36667$$

Let's try a smaller interval:

$$\text{Avg. vel. from } t = 5 \text{ to } t = 5.0001 = \frac{f(5.0001) - f(5)}{5.0001 - 5} \approx 1.333367$$

Estimation: $f'(5) \approx 1.333$. (In fact, $f'(5) = \frac{4}{3}$.)

- (d) From the graph, what do you expect $f'(3)$ to be? Estimate the value of $f'(3)$ numerically using small intervals.

Looking at the graph, the tangent of f at $t = 3$ is a horizontal line. So, we can expect $f'(3) = 0$.

To estimate $f'(3)$, consider the average velocity from $t = 3$ to $t = 3.1$. (You can make other choices for intervals that contain $t = 3$.)

$$\text{Avg. vel. from } t = 3 \text{ to } t = 3.1 = \frac{f(3.1) - f(3)}{3.1 - 3} = \frac{\frac{1}{300} - 0}{.1} \approx .0333333$$

Let's try a smaller interval:

$$\text{Avg. vel. from } t = 3 \text{ to } t = 3.0001 = \frac{f(3.0001) - f(3)}{3.0001 - 3} \approx .00003333$$

Estimation using small intervals: $f'(3) \approx 0$.

- (e) For what values of t is $f'(t)$ positive? Negative? Equal to zero?

Looking at the graph and considering the tangent lines of f for values of t we see that, for $0 < t < 3$, $f'(t)$ is negative. For $t = 3$, $f'(t) = 0$. For $t > 3$, $f'(t)$ is positive.