Math 148

Answers to the Take-Home Portion of the Final Exam

1. Finding the crit. numbers: Solve $f'(x) = 2x^2 - 13x + 15 = 0 \quad \Rightarrow \quad (2x-3)(x-5) = 0$
$\Rightarrow \quad x = 1.5, \quad x = 5$

   The global max or min values will occur at endpoints or critical numbers. Since $x = 5$ is outside of the domain, we only have to evaluate $g(0)$, $g(1.5)$, and $g(4)$.
   $g(0) = -20 \qquad g(1.5) = -9.875 \qquad g(4) \approx -21.3333$

   $\boxed{\text{Max value} = g(1.5) = -9.875, \quad \text{Min value} = g(4) \approx -21.3333}$

2. Current market price $= p = 400 - e^{0.02(200)} = \$345.40/\text{unit}$

   $$\text{Consumer Surplus} = \int_0^{200} 400 - e^{0.02q} - 345.40\, dq = 54.6q - \frac{e^{0.02q}}{0.02} \Big|_0^{200} = \boxed{\$8240.09}$$

3. Holding y constant: $\dfrac{\partial z}{\partial x} = 4xe^y + 5 \quad \Rightarrow \quad \boxed{\dfrac{\partial z}{\partial x}\Big|_{(3,1)} = 4(3)e^1 + 5 \approx 37.6194}$

   Holding x constant: $\dfrac{\partial z}{\partial y} = 2x^2e^y + \dfrac{1}{y} \quad \Rightarrow \quad \boxed{\dfrac{\partial z}{\partial y}\Big|_{(3,1)} = 2(3)^2e^1 + \frac{1}{1} \approx 17.3097}$

4. (a) $f(65, 50) = 18.8$ thousand tons $\Rightarrow$ If the average daily temperature is 65°F and the yearly rainfall is 50 inches, then 18.8 thousand tons of avocados are produced.

   (b) (Here's one way to approximate these, but you could use a left-estimate or a right-estimate. I am using the average of the two.)

   Holding R constant at 50, $\boxed{f_T(65, 50) \approx \dfrac{19.5 - 12.5}{10} = 0.7 \text{ thousand tons/°F}}$

   This means that if the yearly rainfall is 50 inches, and if the average daily temperature were to change from 65°F to 66°F, then the avocado production would increase by approximately 0.7 thousand tons.

   Holding T constant at 65, $\boxed{f_R(65, 50) \approx \dfrac{20.4 - 13.3}{20} = 0.355 \text{ thousand tons/inch}}$

   This means that if the average daily temperature is 65°F, and if the yearly rainfall were to change from 50 to 51 inches, then the avocado production would increase by approximately 0.355 thousand tons.

(c) $f(67, 53)$ represents the avocado production if the average daily temperature is 67°F and the yearly rainfall is 53 inches. Given the information we have about $f(65, 50)$, $f_T(65, 50)$, and $f_R(65, 50)$, we can estimate $f(67, 53)$ since it represents an increase in daily temperature by 2 degrees and an increase in rainfall by 3 inches.

$$f(67, 53) \approx f(65, 50) \; + \; 2 \cdot f_T(65, 50) \; + \; 3 \cdot f_R(65, 50)$$
$$= 18.8 + 2(0.7) + 3(0.355) = \boxed{21.265 \text{ thousand tons}}$$

5. Since $F'(x)$ is positive for $-1 < x < 4$, we know that $F(x)$ is increasing over that interval. Since $\int_{-1}^{4} F'(x)\,dx = 15$, we see that $F(x)$ increased by 15 over that interval.

So, $F(4) = F(-1) + 15 \quad \Rightarrow \quad \boxed{F(-1) = -8}$

Similarly, using the areas between $F'(x)$ and the x-axis, we have that $\boxed{F(1) = -2, \quad F(7) = 2.5, \quad \text{and}}$ $\boxed{F(9) = 2.5.}$

Note that $F(x)$ is increasing for $-1 < x < 4$, decreasing for $4 < x < 7$, and constant for $7 < x < 9$.