Math 148 Quiz #6 Answers

1. Using substitution ($u = 3x+1, \quad du = 3\ dx \ \Rightarrow \ \frac{1}{3}\ du = dx$):

$$\int \frac{6}{3x+1}\,dx \;=\; \int \frac{2}{u}\,du \;=\; 2\ln|u| + C \;=\; 2\ln|3x+1| + C$$

So, $\displaystyle\int_1^2 \frac{6}{3x+1}\,dx = 2\ln|3x+1|\ |_1^2 \;=\; \boxed{2\ln(7) - 2\ln(4) \approx 1.1192}$

2. Note: The graph of this is the same as the parabola x^2 shifted down by 9 units. It has x-intercepts $(\pm 3, 0)$. Given the graph, we can see that we have two regions (One above the x-axis from $x = 3$ to $x = 6$ and one region below the x-axis from $x = -3$ to $x = 3$).

Since $\displaystyle\int_3^6 x^2 - 9\,dx \;=\; \frac{1}{3}x^3 - 9x\ |_3^6 = 36,$ and $\displaystyle\int_{-3}^3 x^2 - 9\,dx \;=\; \frac{1}{3}x^3 - 9x\ |_{-3}^3 = -36,$
we have that the total area of the two regions is $36 + 36 = \boxed{72.}$

(Note: To find $\displaystyle\int_{-3}^3 x^2 - 9\,dx$, you can instead find $\displaystyle\int_0^3 x^2 - 9\,dx$ and double the result.)

3. (a) Looking at the areas of the region above the x-axis and below: $\displaystyle\int_2^{10} f(t)\,dt = 8 - 6 = 2\ ^\circ\text{F}.$

This tells us that $\boxed{\text{the total change in the temperature of the room from time } t = 2 \text{ to } t = 10 \text{ is } 2^\circ\text{F.}}$

(b) Since the rate of change of temperature ($f(t)$) is positive from $0 \le t < 6$, we know that the temperature of the room is increasing for $0 \le t < 6$. Since the rate of change of temperature ($f(t)$) is negative from $6 < t \le 12$, we know that the temperature of the room is decreasing for $6 < t \le 12$. So, the temperature must be at its maximum at $\boxed{t = 6.}$

Since $\displaystyle\int_2^6 f(t)\,dt = 8 =$ total change in temperature between $t = 2$ and $t = 6$, we have that the maximum is $60 + 8 = \boxed{68^\circ\text{F.}}$