Math 148 Quiz #4 Answers

1. (a) $\boxed{f'(x) = e^{-x}(-1)\cdot\ln(3x) + e^{-x}\cdot\frac{1}{3x}(3) \qquad \text{OR} \qquad f'(x) = -e^{-x}\cdot\ln(3x) + \frac{e^{-x}}{x}}$

(Using the product rule and chain rule.)

(b) $\boxed{\frac{dy}{dx} = \frac{(5x^4-x)\cdot 2-(2x+7)(20x^3-1)}{(5x^4-x)^2}}$ (Using the quotient rule.)

2. (a) Finding critical numbers: $g'(x) = -4x^3 + 12x^2 \Rightarrow -4x^3 + 12x^2 = 0$

$$-4x^2(x-3) = 0$$

$$\Rightarrow \boxed{x = 0,\ x = 3}$$

To classify, you can use the first or the second derivative test.

- If you use the first derivative test, you should use a sign chart. You will find that for $x < 0$, $g'(x)$ is positive. For $0 < x < 3$, $g'(x)$ is also positive. For $x > 3$, $g'(x)$ is negative.
Given this information, you can see that **there is a local maximum at $x = 3$, and neither a local maximum nor a local minimum at $x = 0$.**

- If you use the second derivative test, you need the second derivative ($g''(x) = -12x^2 + 24x$). You will find that $g''(3)$ is negative, which tells you that you have a local maximum at $x = 3$. Since $g''(0) = 0$, the second derivative test is inconclusive and you would have to use the first derivative test for $x = 0$.

(b) Possible locations of inflection points: $g''(x) = -12x^2 + 24x \Rightarrow -12x^2 + 24x = 0$

$$-12x(x-2) = 0$$

$$\Rightarrow \boxed{x = 0,\ x = 2}$$

To determine if $g(x)$ switches concavity at these values, we need to see if $g''(x)$ switches sign at these values. To do so, you can use a sign chart for the second derivative. You will find that for $x < 0$, $g''(x)$ is negative. For $0 < x < 2$, $g''(x)$ is positive. For $x > 2$, $g''(x)$ is negative.
Given this information, you can see that there are inflection points at both $x = 0$ and $x = 3$. So, the inflection points are $\boxed{(0, g(0)) = (0, 7) \quad \text{and} \quad (2, g(2)) = (2, 23).}$

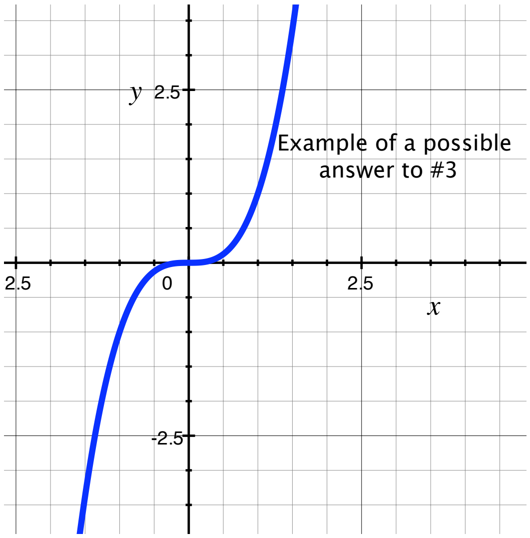
(c) Global max and min values can only occur at the endpoints and the critical numbers. Checking the values of $g(x)$ at $x = -1,\ x = 0,\ x = 3$:

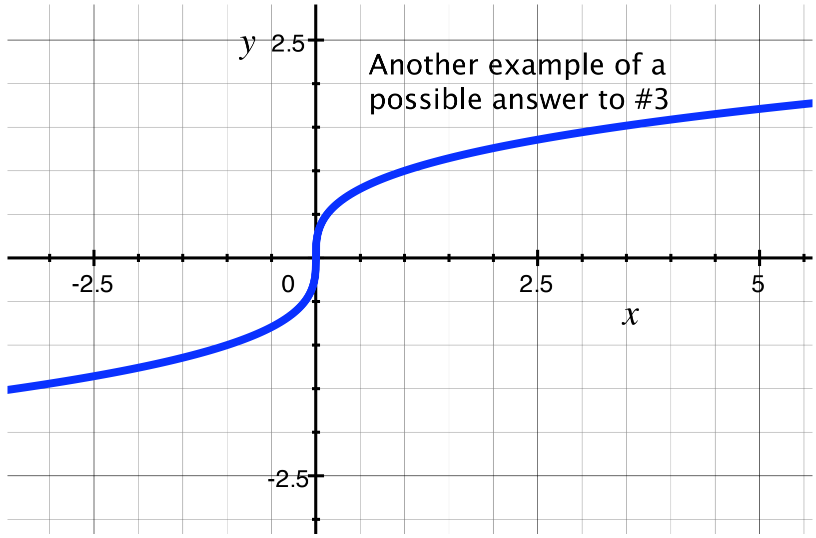
$g(-1) = 2, \qquad g(0) = 7, \qquad g(3) = 34$

So, the **global maximum value is 34** (occurs at $x = 3$), and the **global minimum value is 2** (occurs at $x = -1$).

3. This graph could involve a critical point at which the function has a slope of zero, but the first derivative does not change sign at that point. This means that it will look like a "chair" much like x^3. (Must increase, flatten out, and then increase again OR decrease, flatten out, and then decrease again.)

Alternatively, your graph could involve a critical point at which the function has an "infinite" slope. This will resemble something like $\sqrt[3]{x}$. See below.


y
2.5
Example of a possible
answer to #3
2.5
0
2.5
x
-2.5


y
2.5
Another example of a
possible answer to #3
-2.5
0
2.5
5
x
-2.5