# Math 148
## Exam 2 Answers

1. Note: I am not simplifying these answers, but you could if you choose to.

   (a) $\frac{d}{dx}[\sqrt{4x^2+x} + \frac{1}{x} + x] = \boxed{\frac{1}{2}(4x^2+x)^{-1/2}\cdot(8x+1) - x^{-2} + 1}$ (Chain rule on the first term)

   (b) $\frac{d}{dt}[7e^{5t}\cdot t^{11}] = \boxed{7e^{5t}\cdot 5\cdot t^{11} + 7e^{5t}\cdot 11t^{10}}$ (Product Rule)

   (c) $\frac{d}{ds}\left[\frac{3s^2}{\ln(s)+4}\right] = \boxed{\frac{(\ln(s)+4)\cdot 6s - 3s^2(\frac{1}{s})}{(\ln(s)+4)^2}}$

   (d) $\int \frac{3}{x^5} + 9e^{3x} + 1\ dx = \boxed{-\frac{3}{4}x^{-4} + \frac{9}{3}e^{3x} + x + C}$

   (e) $\int 6t^2(t^3+2)^3\,dt = \int 2(u)^3\,du = \frac{1}{2}u^4 + C = \boxed{\frac{1}{2}(t^3+2)^4 + C}$
   (Using the substitution: $u = t^3+2$, $du = 3t^2\ dt$ or $\frac{1}{3}\ du = t^2\ dt$)

2. (a) $g'(t) = \frac{20}{t} + 2t - 14$ (Note: The domain of $g(t)$ is $t > 0$, so we do not have to consider negative numbers or $t = 0$.)

   To find the critical numbers: $\frac{20}{t} + 2t - 14 = 0 \Rightarrow$
   $$20 + 2t^2 - 14t = 0$$
   $$2(t^2 - 7t + 10) = 0$$
   $$2(t-2)(t-5) = 0$$
   $$\boxed{\text{Crit. Numbers: } t = 2,\ t = 5}$$

   Classifying crit. numbers:

   - If you use the first derivative test, you need to evaluate $g'(t)$ at values around $t = 2$ and $t = 5$ to find where $g'(t)$ is positive or negative. In doing so, you should find that $g(t)$ has a $\boxed{\text{local maximum at } t = 2 \text{ and a local minimum at } t = 5.}$

   - If you use the second derivative test, you need to evaluate $g''(t) = -\frac{20}{t^2} + 2$ at $t = 2$ and $t = 5$ and look at the sign of $g''(2)$ and $g''(5)$. In doing so, you should find that $g(t)$ has a $\boxed{\text{local maximum at } t = 2 \text{ and a local minimum at } t = 5.}$

   (b) Global max and min values can only occur at the endpoints or at the critical numbers of the domain. So, they can occur at $t = 1, t = 4$, or $t = 2$. ($t = 5$ is outside of the domain.)

   Since $g(1) = -13$, $g(4) \approx -12.2741$, and $g(2) \approx -10.1371$, the $\boxed{\text{global maximum value is } g(2) \approx -10.1371, \text{ and the global minimum value is } g(1) = -13.}$

3. (a) $\boxed{\text{You should increase production since the } MR(600) > MC(600).}$
   ($MR(600) = 24$ and $MC(600) = 19$)

(b) We are looking for quantities for which $MR = MC$. Maximum profit will occur after $MR > MC$, which are quantities for which profit is increasing. Since $MR = MC$, when $q = 400$ and for some quantity between 800 and 900, say $q \approx 850$, these are the possible production levels that maximize profit.
Since we switch from $MR > MC$ to $MR < MC$ at $\boxed{q \approx 850,}$ this is the approx. quantity that gives maximum profit.

(c) Since $AC(600) = 23$, the total cost of producing 600 items is
$C(600) = 600 \cdot AC(600) = 600(23) = \boxed{\$13800.}$

(d) Profit $= R(600) - C(600) = 600(24) - 13800 = \boxed{\$600}$

4. $\int_{-2}^{6} f(x)\,dx =$ (Area between $f(x)$ and the $x$-axis above the $x$-axis) $-$ (Area between $f(x)$ and the $x$-axis below the $x$-axis)

So, $\int_{-2}^{6} f(x)\,dx = 9 - 2 = \boxed{7}$

5. (a) Critical numbers: (Looking for where $f'(x) = 0$) $\boxed{x = -1,\ x = 1,\ x = 4}$

Since $f'(x)$ switches from positive to negative at $x = -1$, $f(x)$ must have a $\boxed{\text{local maximum at } x = -1.}$
Since $f'(x)$ switches from negative to positive at $x = 1$, $f(x)$ must have a $\boxed{\text{local minimum at } x = 1.}$
Since $f'(x)$ switches from positive to negative at $x = 4$, $f(x)$ must have a $\boxed{\text{local maximum at } x = 4.}$

(b) We are looking for where $f''(x) = 0$: $x = 0,\ x = 1,\ x = 2.5$ (Note: $f''(x) =$ slope of $f'(x)$)

In order to be the $x$-coordinate of an inflection point, we need $f''(x)$ to switch sign at the $x$-coordinate. Since $f''(x)$ switches sign only at $\boxed{x = 0 \text{ and } x = 2.5,}$ these are the $x$-coordinates of the inflection points of $f(x)$.