Math 148
Exam 1 Solutions

1. (a) $MC(500) = C'(500)$, so we need to estimate the derivative of cost at 500 monkeys. Three possible ways to estimate the derivative (Any of the answers below is acceptable):
   - Using (480,9700) and (500,9940): $MC(500) \approx \frac{9940-9700}{500-480} = \boxed{\text{12 dollars/monkey}}$
   - Using (500,9940) and (520, 10,200): $MC(500) \approx \frac{10{,}200-9940}{520-500} = \boxed{\text{13 dollars/monkey}}$
   - Using (480,9700) and (520,10,200), which is the same as averaging the first two estimates:
   $$MC(500) \approx \frac{10{,}200-9700}{520-480} = \boxed{\text{12.5 dollars/monkey}}$$

   (b) Yes, since it will cost less than \$15 to produce the 501st monkey (see part a). (The company will profit from the 501st monkey.

2. Note that $f(120)$ = the temperature at 2 pm. Using a linear approximation ($\Delta t = 120-112 = 8$), $f(120) \approx f(112)+8f'(112) = 65+8(.25) = 67$. So, the temperature will be approximately 67 degrees at 2 pm.

3. (a) $g'(x) = 4x^3 - 24x^2 + 3$ We are looking for where $g'(x) = 3$
   $$\Rightarrow 4x^3 - 24x^2 + 3 = 3$$
   $$\Rightarrow 4x^3 - 24x^2 = 0$$
   $$\Rightarrow 4x^2(x-6) = 0$$
   $$\Rightarrow \boxed{x = 0 \text{ and } x = 6}$$

   (b) $g''(x) = 12x^2 - 48x$ $g'(-1) = 60$. Since $g''(-1)$ is positive, $g(x)$ is $\boxed{\text{concave up}}$ at $x = -1$.

4. $\frac{d}{dx}[-\ln(x) + \frac{1}{2}e^x + \ln(5)] = -x^{-1} + \frac{1}{2}e^x + 0$ OR $\boxed{-\frac{1}{x} + \frac{1}{2}e^x}$

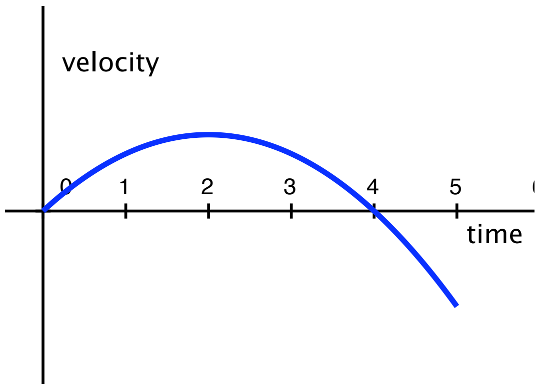
5. (a) At time 0, you have travelled 0 miles. After 3 minutes, you have travelled approximately 4 miles, so the average velocity is $\frac{4-0}{3-4} = \boxed{\frac{4}{3}\text{ miles/minute.}}$

   (b) The instantaneous velocity at a time t is the derivative of the function $f(t)$. Visualizing the slope of the function f, we can approximate that the steepest tangent line occurs around $\boxed{t = 2.25\text{ minutes.}}$

   (c) The acceleration at a time t is the second derivative of the function $f(t)$. We can see the sign of the second derivative by looking at the concavity of the graph f. The acceleration is positive when the graph of f is concave up $\Rightarrow$ The acceleration is positive for $\boxed{0 < t < 2.25\text{ minutes (approximately).}}$

(d) Velocity at time $t = f'(t)$, so we need to graph the derivative of $f(t)$. Note that the derivative (slope) of f is 0 at $t = 0$ and $t = 4$. The derivative (slope) of f is positive for $0 < t < 4$. The derivative (slope) of f is negative for $t > 5$.

So, we have the following rough sketch of the velocity.



6. (a) $h'(x) = x^{-1/2} - 8^{-3} \qquad h'(1) = 1 - 8 = -7$
Since $h'(1)$ is negative, the function $h(x)$ is $\boxed{\text{decreasing}}$ at $x = 1$.

(b) To find the equation of the tangent line, we need the slope of the line and a point on the line.

Slope $= h'(1) = -7$ (from part (a))

We know that the tangent line is tangent to the graph of h at the point $(1, h(1)) = (1, 6)$

Using the slope-intercept form of a line, we have that the equation of the tangent line is $\boxed{y - 6 = -7(x - 1) \quad \text{or} \quad y = -7x + 13.}$

7. (a) Since $f'(t) = \frac{dC}{dt}$ = the rate of change of cars at a time t (in hours), the units are $\boxed{\text{cars/hour.}}$

(b) Since $f'(t)$ is positive for $0 < t < 2$ and $7 < t < 10$, we know that $f(t)$ is increasing for $\boxed{0 < t < 2 \text{ and } 7 < t < 10.}$