

## Math 120 Quiz #9 Solutions

1. (5 pts.) A boater heads in the direction N  $30^\circ$  W with a speed of 40 mph relative to the water (see below). If the water is flowing directly west at 6 mph, what is the true velocity (vector) and true speed of the boat?

Let  $\mathbf{v}$  be the velocity of the boat relative to the water. This vector has direction  $60^\circ$  (since it is  $30^\circ$  west of north) and length 40.

$$\begin{aligned}\text{We can write } \mathbf{v} \text{ in component form: } \mathbf{v} &= \langle 40 \cos 120^\circ, 40 \sin 120^\circ \rangle \\ &= \langle -20, 20\sqrt{3} \rangle\end{aligned}$$

Let  $\mathbf{w}$  be the velocity of the water current. Then  $\mathbf{w} = \langle -6, 0 \rangle$  since it flows directly east.

Note that the true velocity will be the sum of the velocity  $\mathbf{v}$  of the boater and the velocity  $\mathbf{w}$  of the water.  
 $\Rightarrow \mathbf{v}_{\text{true}} = \mathbf{v} + \mathbf{w} = \langle -26, 20\sqrt{3} \rangle.$

The true speed is given by  $|\mathbf{v}_{\text{true}}| = \sqrt{(-26)^2 + (20\sqrt{3})^2} = \sqrt{1876} \approx 43.3128$  mph.

2. (5 pts.) Find the angle between the vectors  $\mathbf{u} = \langle 3, 4 \rangle$  and  $\mathbf{v} = \langle 1, -4 \rangle$ .

We will use the following property of the dot product:  $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos \theta$  where  $\theta$  is the angle between  $\mathbf{u}$  and  $\mathbf{v}$  such that  $0 \leq \theta \leq \pi$

$$\text{Calculations: } \mathbf{u} \cdot \mathbf{v} = 3 \cdot 1 + 4 \cdot (-4) = -13$$

$$|\mathbf{u}| = \sqrt{3^2 + 4^2} = 5$$

$$|\mathbf{v}| = \sqrt{1^2 + (-4)^2} = \sqrt{17}$$

$$\text{So, } -13 = 5\sqrt{17} \cos \theta \quad \Rightarrow \quad \cos \theta = \frac{-13}{5\sqrt{17}} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{-13}{5\sqrt{17}}\right) \approx 2.2531 \text{ radians or } 129.0939^\circ$$