Math 120 Quiz #6 Solutions

1. (3 pts.) Find the exact value of $\sin(\frac{7\pi}{12})$.

   You can use the angle addition formula for sine or the half-angle formula to get this exact value.

   Using the angle addition formula:
   $$\sin(\tfrac{7\pi}{12}) = \sin(\tfrac{\pi}{4} + \tfrac{\pi}{3})$$
   $$= \sin(\tfrac{\pi}{4})\cos(\tfrac{\pi}{3}) + \cos(\tfrac{\pi}{4})\sin(\tfrac{\pi}{3})$$
   $$= \tfrac{\sqrt{2}}{2}\cdot\tfrac{1}{2} + \tfrac{\sqrt{2}}{2}\cdot\tfrac{\sqrt{3}}{2}$$
   $$= \tfrac{\sqrt{2}+\sqrt{6}}{4}$$

   Using the half-angle formula:
   $$\sin(\tfrac{7\pi}{12}) = \pm\sqrt{\tfrac{1}{2}(1-\cos(\tfrac{7\pi}{6}))}$$
   $$= \pm\sqrt{\tfrac{1}{2}(1-(-\tfrac{\sqrt{3}}{2}))}$$
   $$= \pm\sqrt{\tfrac{1}{2}(1+\tfrac{\sqrt{3}}{2})}$$

   Since $\frac{7\pi}{12}$ is in quadrant II, sine is positive. So $\sin(\frac{7\pi}{12}) = \sqrt{\frac{1}{2}(1+\frac{\sqrt{3}}{2})}$.

2. (3 pts.) If x is in quadrant I and $\sin\ x = \frac{8}{17}$, find the exact value of $\sin(2x)$.

   Note that since $\sin\ x = \frac{8}{17}$, $\cos\ x = \pm\frac{15}{17}$. (Using a reference triangle or Pythagorean theorem.)

   Since x is in quadrant I, cosine is positive. So, $\cos\ x = \frac{15}{17}$.

   $$\Rightarrow \quad \sin(2x) = 2\sin\ x\cos\ x = 2\cdot\tfrac{8}{17}\cdot\tfrac{15}{17} = \tfrac{240}{289}.$$

3. (2 pts.) If θ is in quadrant IV and $\cos\ \theta = \frac{5}{8}$, find the exact value of $\sin(\frac{\theta}{2})$.

   $$\sin(\tfrac{\theta}{2}) = \pm\sqrt{\tfrac{1}{2}(1-\cos\ \theta)} = \pm\sqrt{\tfrac{1}{2}(1-\tfrac{5}{8})} = \pm\sqrt{\tfrac{1}{2}(\tfrac{3}{8})} = \pm\sqrt{\tfrac{3}{16}} = \pm\tfrac{\sqrt{3}}{4}$$

   Note that if θ is in quadrant IV, $\frac{\theta}{2}$ is in quadrant II. So, $\sin(\frac{\theta}{2})$ is positive. $\Rightarrow \sin(\frac{\theta}{2}) = \frac{\sqrt{3}}{4}$.

4. (2 pts.) Find the exact values of the following:

   (a) $\tan^{-1}(-1)$

   Note that $\tan^{-1}(-1)$ will give you an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is -1.

   Since $\tan(-\frac{\pi}{4}) = -1$, we have that $\tan^{-1}(-1) = \frac{\pi}{4}$.

   (b) $\cos^{-1}(\cos(\frac{7\pi}{6}))$

Note that $\cos(\frac{7\pi}{6}) = -\frac{\sqrt{3}}{2}$.

So, $\cos^{-1}(\cos(\frac{7\pi}{6})) = \cos^{-1}(-\frac{\sqrt{3}}{2})$ = the angle between 0 and π whose cosine is $-\frac{\sqrt{3}}{2}$.

Since $\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$, we have that $\cos^{-1}(\cos(\frac{7\pi}{6})) = \cos^{-1}(-\frac{\sqrt{3}}{2}) = \frac{5\pi}{6}$.