Math 120 Quiz #5 Solutions

1. (5 pts.) You board a ferris wheel that makes one complete revolution every 30 seconds. At time 0, you begin at the lowest point of the ferris wheel which is 6 feet above the ground. The highest point of the wheel is 86 feet above the ground.
Find an equation that gives your height above the ground at time t, given that the height varies sinusoidally.

The maximum height is 86 feet and the minimum height is 6 feet. This means that the amplitude is given by $\frac{1}{2}[86-6]=40$ and the mean is given by $\frac{1}{2}[86+6]=46$.

The period is 30 seconds. So, $\omega=\dfrac{2\pi}{30}=\dfrac{\pi}{15}$.

If using cosine, here is one possible equation for the height:

$$h(t)=40\cos\tfrac{\pi}{15}(t-15)+46$$

Here are **some** other possible answers.

$h(t)=-40\cos\tfrac{\pi}{15}t+46$ $\qquad$ $h(t)=40\sin\tfrac{\pi}{15}(t-7.5)+46$

2. (5 pts.) Prove the following trigonometric identity: $\dfrac{\sec x+\csc x}{\tan x+\cot x}=\sin x+\cos x$
Here are a couple of ways to prove this identity:

- Convert to sines and cosines:

$$\frac{\sec x+\csc x}{\tan x+\cot x}=\frac{\frac{1}{\cos x}+\frac{1}{\sin x}}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}$$

Getting a common denominator in both the numerator and the denominator yields the following:

$$\begin{aligned}\frac{\frac{\sin x+\cos x}{\sin x\cdot\cos x}}{\frac{\sin^2x+\cos^2x}{\sin x\cdot\cos x}}&=\frac{\sin x+\cos x}{\sin x\cdot\cos x}\cdot\frac{\sin x\cdot\cos x}{\sin^2x+\cos^2x}\\&=\frac{\sin x+\cos x}{\sin^2x+\cos^2x}\\&=\sin x+\cos x\end{aligned}$$

- Multiply the numerator and denominator by $\tan x$. (Note that $\tan x\cdot\cot x=1$.)

$$\begin{aligned}\frac{\sec x+\csc x}{\tan x+\cot x}\cdot\frac{\tan x}{\tan x}&=\frac{\sec x\cdot\tan x+\csc x\cdot\tan x}{\tan^2x+\cot x\cdot\tan x}\\&=\frac{\sec x\cdot\tan x+\sec x}{\tan^2x+1}\\&=\frac{\sec x\cdot\tan x+\sec x}{\sec^2x}\\&=\frac{\tan x+1}{\sec x}\\&=(\tan x+1)\cdot\cos x\\&=\sin x+\cos x\end{aligned}$$

Bonus Problem (1 pt. extra credit):
Find the exact value of $\sin\dfrac{5\pi}{3}$.

The reference number for $\frac{5\pi}{3}$ is $\frac{\pi}{3}$.

Since $\frac{5\pi}{3}$ is in quadrant IV, sine is negative. So, $\sin\dfrac{5\pi}{3}=-\sin\dfrac{\pi}{3}=-\dfrac{\sqrt{3}}{2}$.