Math 120 Quiz #3 Solutions

1. (3 pts.) Two straight roads diverge at an angle of 50^o. Two cars leave the intersection at the same moment. One car travels at 40 mph and the other at 60 mph. How far apart are the cars after 45 minutes?

   After 45 minutes, one car has traveled $d_1 = r_1 t = (60)\frac{45}{60} = 45$ miles and the other has traveled $d_2 = r_2 t = (40)\frac{45}{60} = 30$ miles.

   To solve for $x =$ the distance between the cars, we must use the Law of Cosines.

   $$x^2 = 30^2 + 45^2 - 2(30)(45)\cos 50^o \approx 1189.4735$$

   $$\Rightarrow \quad x \approx 34.3887 \text{ miles}$$

2. (4 pts.) Find the terminal point $P(x, y)$ on the unit circle determined by $t = \frac{11\pi}{6}$. (Give **exact** values of x and y.)

   Note that the terminal point of t lies in quadrant IV and the reference number is $\bar{t} = \frac{\pi}{6}$.

   So, the terminal point will be given by $P(x, y) = (\cos\frac{11\pi}{6}, \sin\frac{11\pi}{6}) = (\cos\frac{\pi}{6}, -\sin\frac{\pi}{6})$
   $= (\frac{\sqrt{3}}{2}, -\frac{1}{2})$

3. (3 pts.) Find the **exact** values of the trigonometric functions of t given that $\tan t = -\frac{5}{3}$ with the terminal point of t in quadrant II.

   To find the values, you can draw a reference triangle with opposite side of length 5 and adjacent side of length 3 and consider which trigonometric ratios are positive or negative in quadrant II.

   For fun, here is a way to find the values using the Pythagorean Identities.

   - Since $\cot t = \frac{1}{\tan t}$, $\quad \cot t = -\frac{3}{5}$.
   - If $\tan t = -\frac{5}{3}$, then $\quad (-\frac{5}{3})^2 + 1 = \sec^2 t \qquad \Rightarrow \qquad \sec t = \pm\frac{\sqrt{34}}{3}$.

     In quadrant II, secant is negative, so $\quad \sec t = -\frac{\sqrt{34}}{3}$.
   - Since $\sec t = \frac{1}{\cos t}$, $\quad \cos t = -\frac{3}{\sqrt{34}}$.
   - If $\cos t = -\frac{3}{\sqrt{34}}$, then $\quad (-\frac{3}{\sqrt{34}})^2 + \sin^2 t = 1 \qquad \Rightarrow \qquad \sin t = \pm\frac{5}{\sqrt{34}}$.

     In quadrant II, sine is positive, so $\quad \sin t = \frac{5}{\sqrt{34}}$.
   - Since $\csc t = \frac{1}{\sin t}$, $\quad \csc t = \frac{\sqrt{34}}{5}$.