Math 120
Exam 1 Solutions

1. A person is riding a ferris wheel with a radius of 30 feet. The wheel rotates once every two minutes.

   (a) Find the distance along the arc that the rider travels in 30 seconds.

   Since the wheel rotates twice every two minutes, the ferris wheel will make a quarter of a rotation in 30 seconds. (30 seconds is $\frac{1}{4}$th of 2 minutes.)

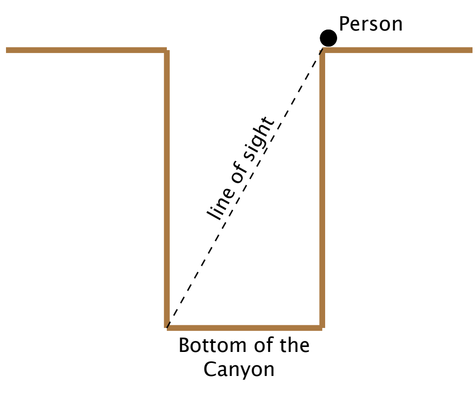
   Two ways to calculate the distance:

   - The circumference of the wheel is $C = 2\pi(30) = 60\pi$ feet.
     $\Rightarrow$ The distance traveled is $\frac{1}{4}C = 15\pi$ feet.
   - The distance along the arc subtends the angle $\theta = \dfrac{\pi}{2}$. Using the formula for the length of a circular arc, we have that $s = 30(\dfrac{\pi}{2}) = 15\pi$ feet.

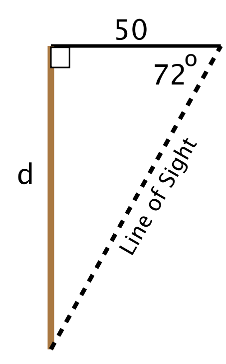
   (b) Find the angular and linear speeds of the rider. (Include units.)

   - Angular Speed ω: The wheel traverses 2π radians in 2 minutes, so the angular speed is $\omega = \dfrac{2\pi}{2} = \pi$ radians per minute.
   - Linear Speed v: You can do a similar calculation as for angular speed using the circumference of the wheel or since we have the angular speed, we can use the fact that $v = r\omega$.
     $\Rightarrow$ $v = 30\pi$ feet per minute.

2. A person stands at the edge of a straight-walled canyon and measures the angle of depression to the bottom of the other side of the canyon to be 72^o. If the distance across the canyon is 50 meters, how deep is the canyon? (Ignore the height of the person.)



The information above gives us the following right triangle.



If d = depth of the canyon, then we have that $\tan 72^o = \dfrac{d}{50}$ $\Rightarrow$ $d = 50 \tan 72^o$
≈ 153.8842 meters

3. (a) (5 pts.) Find the reference number of $t = \dfrac{32\pi}{3}$.

Note that $\dfrac{32\pi}{3} = \dfrac{30\pi}{3} + \dfrac{2\pi}{3} = 10\pi + \dfrac{2\pi}{3}$.

Since the angle $\dfrac{2\pi}{3}$ lies in quadrant II, t is in quadrant II.

The reference number is given by $\bar{t} = 11\pi - \dfrac{32\pi}{3} = \dfrac{\pi}{3}$.

(b) (6 pts.) Find the point on the unit circle determined by $t = \dfrac{32\pi}{3}$.

The point on the unit circle is given by $(\cos\dfrac{32\pi}{3}, \sin\dfrac{32\pi}{3}) = (-\cos\dfrac{\pi}{3}, \sin\dfrac{\pi}{3}) = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$ since $\dfrac{32\pi}{3}$ is in quadrant II.

(c) Find the exact value of $\tan\dfrac{32\pi}{3}$.

Two ways to calculate:

- Since tangent is negative in quadrant II, $\tan\dfrac{32\pi}{3} = -\tan\dfrac{\pi}{3} = -\sqrt{3}$.
- Using the information from part (b), we have that $\tan\dfrac{32\pi}{3} = \dfrac{\sin\frac{32\pi}{3}}{\cos\frac{32\pi}{3}} = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$.

4. (4 pts.) In what quadrant does the angle $\alpha = 13$ radians lie?

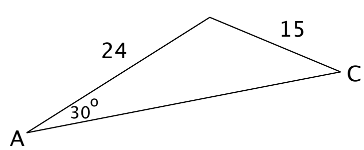
Here are a couple of ways to approach this problem:

- Note that $\dfrac{13}{\pi} \approx 4.1380 \quad \Rightarrow \quad 13 \approx 4.1380\pi$.

  So, 13 is between 4π and 4.5π radians. This places the angle 13 radians in quadrant I.

- Note that 13 radians $\approx 744.8451^\circ$. Since $744.8451^\circ - 720^\circ = 44.8451^\circ$, the angle is in quadrant I.

5. Given that $\angle A = 30^\circ$, $a = 15$, $c = 24$, find **all possible** values of the angle C (See tentative sketch below).



Using the Law of Sines, we have that $\dfrac{\sin\ 30^\circ}{15} = \dfrac{\sin\ C}{24}$

$$\Rightarrow \quad \sin C = \frac{24 \cdot \sin\ 30^\circ}{15} = \frac{24(\frac{1}{2})}{15} = \frac{12}{15} = .8$$

Using the $\sin^{-1}$ key on the calculator, we get that $\angle C \approx 53.1301^{\circ}$.

There is the additional possibility that $\angle C \approx 180^{\circ} - 53.1301^{\circ} = 126.8699^{\circ}$.

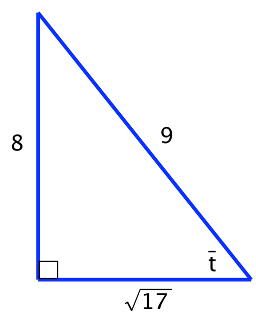
In either possibility, we have that $\angle A + \angle C < 180^{\circ}$.

Answer: 53.1301° and 126.8699°

6. Find the values of the trigonometric functions of t given that $\sin t = \dfrac{8}{9}$ and $\cos t < 0$.

Here are two ways to find the values of the other functions.

- Reference triangle:



The third side of the triangle has length $\sqrt{9^2 - 8^2} = \sqrt{17}$.

Since cosine is negative, $\cos t = -\dfrac{\sqrt{17}}{9}$.

Since cosine is negative and sine is positive, tangent is negative. $\Rightarrow \tan t = -\dfrac{8}{\sqrt{17}}$

Also, $\csc t = \dfrac{9}{8}$, $\sec t = -\dfrac{9}{\sqrt{17}}$, $\cot t = -\dfrac{\sqrt{17}}{8}$

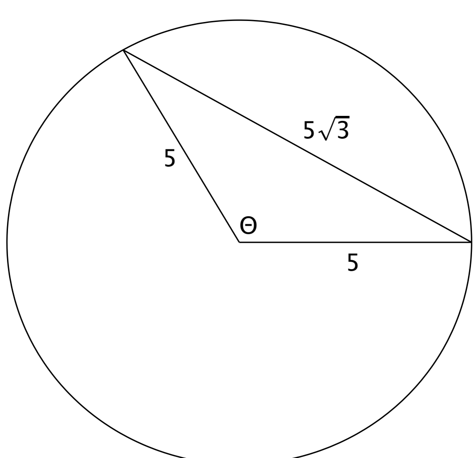
- Using the Pythagorean Identity, we have that $(\frac{8}{9})^2 + \cos^2 t = 1$.

$$\Rightarrow \cos t = \pm\frac{\sqrt{17}}{9}$$

Since cosine is negative, we have that $\cos t = -\dfrac{\sqrt{17}}{9} \quad \Rightarrow \quad \sec t = -\dfrac{9}{\sqrt{17}}$

You can use the other Pythagorean Identities or the Reciprocal Identities to get the other values of $\csc t$, $\cot t$, and $\tan t$.

7. Find the following given the figure below.



(a) The angle θ

One way to find θ is to use the Law of Cosines.

$$(5\sqrt{3})^2 = 5^2 + 5^2 - 2(5)(5)\cos\theta$$
$$75 = 50 - 50\cos\theta \qquad \Rightarrow \qquad \cos\theta = -\frac{1}{2}$$

Using the table for special values of trigonometric ratios or using the $\cos^{-1}$ button on a calculator to find $\cos^{-1}(-\frac{1}{2})$, will yield that $\theta = 120^{\circ}$. (Given that $0 < \theta < 180^{\circ}$.)

(b) The area of the shaded region

The area of the triangle is $A_1 = \frac{1}{2}(5)(5)\sin 120^{\circ} = \frac{25}{2}\left(\frac{\sqrt{3}}{2}\right) = \frac{25\sqrt{3}}{4} \approx 10.8253$.
(Using the formula $A_{\text{triangle}} = \frac{1}{2}ab\sin\theta$.)

Note: $\theta = 120^{\circ} = \dfrac{2\pi}{3}$ radians.

So, the area of the sector is $A_2 = \frac{1}{2}(5)^2\left(\frac{2\pi}{3}\right) = \frac{25\pi}{3} \approx 26.1799$
(Using the formula $A_{\text{sector}} = \frac{1}{2}r^2\theta$.)

The area of the shaded region is $A_2 - A_1 = \frac{25\pi}{3} - \frac{25\sqrt{3}}{4} \approx 15.3546$.