Math 111
Exam 2 Answers

1. (a) $P(3) = 8e^{0.4(3)} \approx 25.5609$ thousand monkeys $\Rightarrow$ $\boxed{\text{25,561 monkeys}}$

   (b) Solving for t: $40 = 8e^{0.4t} \quad \Rightarrow \quad 5 = e^{0.4t}$

   Using natural log: $\boxed{t = \dfrac{\ln 5}{0.4} \approx 4.0236 \text{ years}}$

   (Note: If you used common log: $t = \dfrac{\log 5}{0.4 \log e}$)

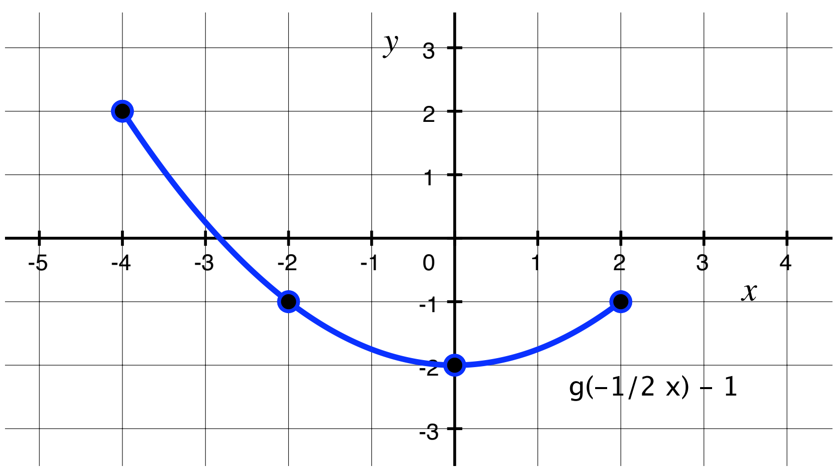
   (c) The population is $\boxed{\text{increasing}}$ each year. Since $e^{0.4} \approx 1.4198$, the population is growing by approx. $\boxed{\text{49.18\% each year.}}$

2. For $\ln(x+2)$ to be defined, we need $x + 2 > 0$. So, the domain is $\boxed{x > -2 \quad \text{or} \quad (-2, \infty).}$

   The vertical asymptote is $\boxed{x = -2}$ since this is the same as $\ln x$ vertically stretched by a factor of 3 and shifted left by 2 units.

3. (a) Domain: $\boxed{-1 \le x \le 2 \quad \text{or} \quad [-1, 2]}$ Range: $\boxed{-1 \le y \le 3 \quad \text{or} \quad [-1, 3]}$

   (b) The transformations of $g(x)$ are:
   $\boxed{\text{Reflect over the } y\text{-axis, Horizontally stretch by a factor of 2, Shift down 1 unit}}$



   (c) Since $3g(x+5)$ is the same as $g(x)$ vertically stretched by 3 and shifted left by 5, the range of $3g(x+5)$ is given by $\boxed{-3 \le y \le 9 \quad \text{or} \quad [-3, 9].}$ (The range is affected only by the vertical stretch.)

4. (a) $\log_x 1331 = 3 \quad \Rightarrow \quad x^3 = 1331 \quad x = \sqrt[3]{1331} = \boxed{11}$

(b) Taking the natural log of both sides: $\ln(2^x) = \ln(0.25(3)^x)$

Using log properties: $x \ln 2 = \ln 0.25 + x \ln 3$

Subtracting $x \ln 3$ from both sides: $x \ln 2 - x \ln 3 = \ln 0.25$

Factoring x from the left side: $x(\ln 2 - \ln 3) = \ln 0.25$ $\quad x = \boxed{\frac{\ln 0.25}{\ln 2 - \ln 3}}$

(Note: If you used common log instead, you would get $x = \frac{\log 0.25}{\log 2 - \log 3}$.)

5. Since the height function is quadratic with a negative a value, the graph is a parabola opening down. So, the maximum height will occur at the vertex.

Using the vertex formula, the t-coordinate of the vertex is $t = \dfrac{-48}{2(-16)} = 1.5$ seconds.

So, the maximum height occurs at $\boxed{\text{1.5 seconds}}$ and is $h(1.5) = \boxed{\text{36 feet.}}$

6. Using the vertex form of a quadratic function: $y = a(x-2)^2 - 5$.

To solve for a, plug in $x = 0$, and $y = 1$: $1 = a(0-2)^2 - 5 \quad \Rightarrow \quad a = \dfrac{3}{2}$

Equation: $\boxed{y = \frac{3}{2}(x-2)^2 - 5}$

7. Need: Two points on the graph of $f(x)$ Have: One point (0,16)

To find the other point, note that since it is on the line, the coordinates have to satisfy the linear equation. When $y = 9$, we can use the linear equation to find that $x = 2$. So, the second point is (2,9).

Since the function is exponential, we know it has the form $f(x) = ab^x$. Since it goes through the point (0,16), we know that $a = 16$. Plugging in $x = 2, y = 9$ gives us that $b = \dfrac{3}{4}$.

Equation: $\boxed{f(x) = 16(\frac{3}{4})^x}$