Math 111
Final Exam Solutions

1. (a) $F(x) > 0$ when the y-coordinates of the points of $F(x)$ are positive (above the x-axis). So, $F(x) > 0$ when $\boxed{x > 4.}$

   (b) Since $y = F(x)$, we have that $x = F^{-1}(y)$. This means that we are looking for the x-value that gives us $y = 1$ on the graph of $F(x)$. This looks like it is $x \approx 4.8$. So, $\boxed{F^{-1}(1) \approx 4.8.}$

   (c) Average rate of change $= \frac{F(4)-F(2)}{4-2} = \frac{0-(-4)}{2} = \boxed{2}$

   (Note: This is the same as the slope between the points $(2, -4)$ and $(4, 0)$.)

2. To find the equation of the line, we need a point on the line and the slope of the line.

   Point: Since the x-intercept is at $x = 8$, the point $(8, 0)$ is on the line.

   Slope: Note that the line $2x + 4y = 12$ can be rewritten as $y = -\frac{1}{2}x + 3$ by solving for y. So, the slope of the line $2x + 4y = 12$ is $m = -\frac{1}{2}$.

   Since the line that we are writing an equation for is parallel to the given line, the slope of the line we are finding an equation for is also $-\frac{1}{2}$.

   Equation of the Line: $y - 0 = -\frac{1}{2}(x - 8) \quad \Rightarrow \quad \boxed{y = -\frac{1}{2}x + 4}$

3. (a) The domain of $f(x)$ is $\boxed{\text{all real numbers.}}$

   (b) The vertex of $f(x) = x^2 + 4$ is $\boxed{(0, 4)}$ since this graph is the same as x^2 shifted up 4 units.

   Note: You can also use the vertex formula to find the vertex.

   (c) **Transformations:** To obtain the graph of $4f(x - 1)$, we can take the graph of $f(x)$ and $\boxed{\text{shift it 1 unit to the right and stretch it vertically by 4.}}$

   Vertex: Shifting the point $(0, 4)$ to the right by $1 \longrightarrow (1, 4)$
   Vertically stretching by $4 \longrightarrow (1, 16)$

   So, the vertex of $4f(x - 1)$ is the point $\boxed{(1, 16).}$

   (d)
$$\begin{aligned} 4f(x-1) &= 4[(x-1)^2 + 4] \\ &= 4[x^2 - 2x + 1 + 4] \\ &= 4[x^2 - 2x + 5] \\ &= \boxed{4x^2 - 8x + 20} \end{aligned}$$

4. (a) The function $h(x)$ will be undefined when the denominator $x-3$ is equal to zero. $\Rightarrow \quad x=3$.

So, the domain of $h(x)$ is $\boxed{\text{all real numbers except } x=3,}$ which can also be written as $\boxed{x \neq 3,}$ or $\boxed{(-\infty,3)\cup(3,\infty).}$

(b) We want to find x such that $\frac{2x^3-6x^2-8x}{x-3}=0 \quad \Rightarrow \quad 2x^3-6x^2-8x=0.$

$$\Rightarrow \quad 2x(x^2-3x-4)=0$$
$$\Rightarrow \quad 2x(x-4)(x+1)=0$$

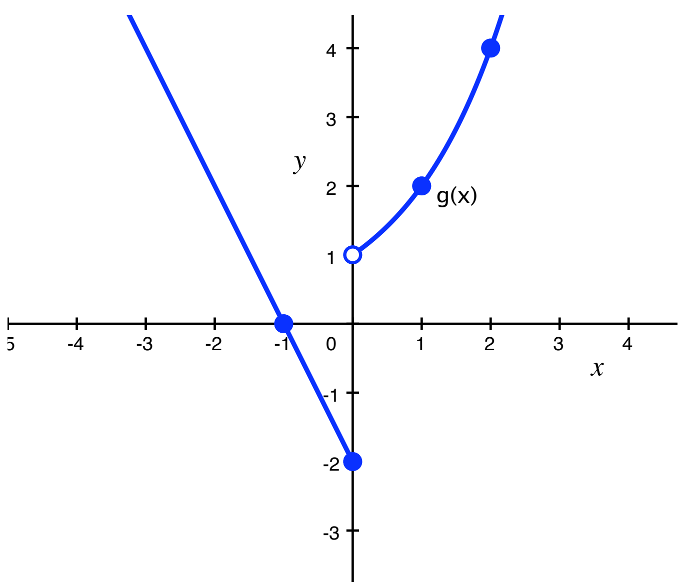
So, $h(x)$ will have zeros at $\boxed{x=0, \quad x=4, \quad \text{and } x=-1.}$

(c) **Horizontal Asymptotes:** In the long-run, $h(x)\approx \dfrac{2x^3}{x}=2x^2$. Since this increases infinitely as $x\to\infty$ or as $x\to-\infty$, we $h(x)$ has $\boxed{\text{no horizontal asymptotes.}}$

Vertical Asymptotes: Note that $\frac{2x^3-6x^2-8x}{x-3}=\frac{2x(x-4)(x+1)}{x-3}$ is in lowest terms, meaning that no common factors can be divided out. Since we have division by 0 at $x=3$, $h(x)$ has a $\boxed{\text{vertical asymptote of } x=3.}$

5. (a) For $x\leq 0$, $g(x)$ is given by $-2x-2$, which is a line with y-intercept (0,-2) and slope -2.

For $x>0$, $g(x)$ is given by 2^x, which is an exponential function through the points (1,2) and (2,4).



(b) From the graph we can see that the range is $\boxed{y\geq -2 \quad \text{or} \quad [-2,\infty).}$

(c) Note that $g(0)=-2(0)-2=-2$ since $x=0$ is less than or equal to 0 (so we are using the first formula).

Also note that $g(1) = 2^1 = 2$ since $x = 1$ is greater than 0 (using the second formula).

Thus, $g(0) + 2g(1) = -2 + 2(2) = \boxed{2.}$

6. To find the points with a y-coordinate of 1, solve $H(x) = 1$

$$\Rightarrow \quad 4x^{1/5} + 9 = 1$$
$$\Rightarrow \quad 4x^{1/5} = -8$$
$$\Rightarrow \quad x^{1/5} = -2$$
$$\Rightarrow \quad x = (-2)^5$$
$$\Rightarrow \quad x = -32$$

So, there is only one point: $\boxed{(-32, 1).}$

7. (a) Using the compound interest formula with $A =?$, $P = 3000$, $r = 0.07$, $k = 4$, $t = 6$:

$$A = 3000(1 + \tfrac{0.07}{4})^{4(6)} = 3000(1.0175)^{24} \approx \$4549.33$$

So, you will have $\boxed{\$4549.33}$ after 6 years.

(b) Note that $(1 + \frac{0.07}{4})^4 \approx 1.071859$. (This is the growth factor after one year.)

So, the effective rate is $(1 + \frac{0.07}{4})^4 - 1 \approx 1.071859 - 1 = 0.071859$ or $\boxed{7.1859\%.}$

8. Using the continuously compounding interest formula with
$A = 3P$, $P =$ initial value, $r = 0.3$, $t = ?$:

$$3P = Pe^{0.3t} \quad \Rightarrow \quad 3 = e^{0.3t} \quad \Rightarrow \quad \ln 3 = 0.3t \ln e$$
$$\Rightarrow \quad \ln 3 = 0.3t \quad \Rightarrow \quad \boxed{t = \tfrac{\ln 3}{0.3}} \approx 3.662$$

So, it would take approximately $\boxed{3.662 \text{ years}}$ for your money to triple.

9. Using the future-value of an annuity formula with $S =?$, $R = 300$, $r = 0.03$, $k = 12$, $t = 4$ years

$$S = 300\left[\frac{(1+\frac{0.03}{12})^{12(4)}-1}{\frac{0.03}{12}}\right] = 300\left[\frac{(1.0025)^{48}-1}{0.0025}\right] \approx \boxed{\$15{,}279.36}$$

10. To find the inverse function, solve for x: $y = \dfrac{5}{x^3}$

$$\Rightarrow \quad x^3 y = 5$$
$$x^3 = \frac{5}{y}$$
$$x = \sqrt[3]{\frac{5}{y}}$$

So, $\boxed{x = f^{-1}(y) = \sqrt[3]{\frac{5}{y}} \quad \text{OR} \quad y = (\frac{5}{y})^{1/3}.}$