Math 111
Exam 2 Solutions

1. (a) Since $P(0) = 50,000$, the initial population is $\boxed{50{,}000}$. (The initial value of an exponential function $a \cdot b^x$ is the value a.)

   (b) Since the growth factor $b = 0.97$, the population is $\boxed{\text{decreasing by } 3\%}$ each year.

   (c) We need to solve for t such that $40000 = 50000(0.97)^t$.
   $$\Rightarrow \quad 0.8 = 0.97^t$$

   Taking the natural log (or common log) of both sides: $\ln 0.8 = t \ln 0.97$
   $$\Rightarrow \quad t = \boxed{\frac{\ln 0.8}{\ln 0.97} \approx 7.326 \text{ years}}$$

2. Given the definition of logs, we have that $5^3 = 3x + 2 \quad \Rightarrow \quad 125 = 3x + 2 \quad \Rightarrow \quad \boxed{x = 41}$

3. If you invest P dollars into an account giving 5% interest each year, then the amount in the account after t years is given by $P(1.05)^t$.

   We want to find the time at which the amount is $2P$. $\Rightarrow$
   $$2P = P(1.05)^t$$
   $$2 = (1.05)^t$$

   Taking the common log (or natural log) of both sides: $\log 2 = t \log 1.05$
   $$\Rightarrow \quad t = \frac{\log 2}{\log 1.05} \approx 14.2067 \text{ years}$$

   So, it will take approximately $\boxed{14.2067 \text{ years}}$ for a certain amount to double.

4. Exponential Function: $y = a \cdot b^x$

   Point (0,8) $\rightarrow \quad a = 8 \qquad (-1, 20) \rightarrow \quad 20 = 8b^{-1} \quad \text{OR} \quad 20 = \frac{8}{b} \quad \Rightarrow \quad b = \frac{2}{5}$

   Equation: $\boxed{y = 8(\tfrac{2}{5})^x}$

5. Here are some possible answers:

   - $\boxed{v(x) = 2 + x^3, \quad u(x) = e^x}$
   - $\boxed{v(x) = x^3, \quad u(x) = e^{2+x}}$

6. Since the function $F(x)$ has zeros at $x = -1$ and $x = 4$, it must have the form $F(x) = a(x+1)(x-4)$.

   To find a, we can use the point $(0. -2)$. $\quad \Rightarrow \quad -2 = a(0+1)(0-4) = -4a \quad \Rightarrow \quad a = \frac{1}{2}$

So, $\boxed{F(x) = \frac{1}{2}(x+1)(x-4).}$

7. (a) Using the vertex formula:

   x-coordinate: $\quad x = -\dfrac{b}{2a} = -\dfrac{24}{2(-3)} = 4$

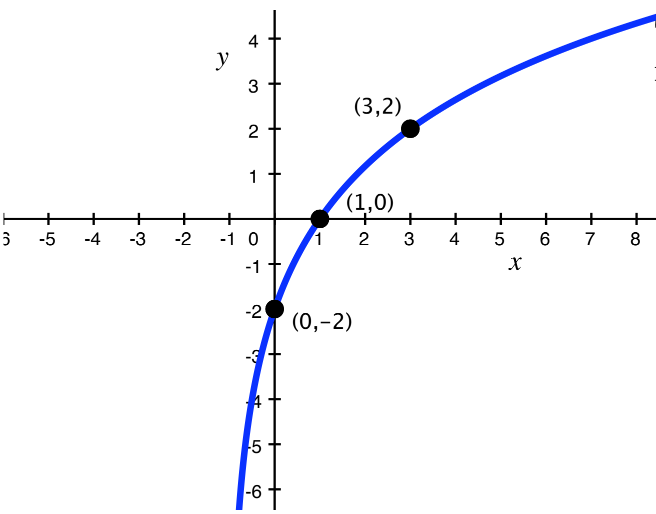
   y-coordinate: $\quad y = h(4) = -3(4)^2 + 24(4) + 1 = 49$

   So, the vertex is the point $\boxed{(4, 49).}$

   (b) Note that since the coefficient of x^2 is negative, the graph of $h(x)$ is a parabola that opens down. So, given the answer to part (a), the range of $h(x)$ is $\boxed{y \leq 49 \ \text{ or } (-\infty, 49].}$

8. (a) The domain of $f(x)$ is $\boxed{x > 2 \text{ or } (2, \infty).}$

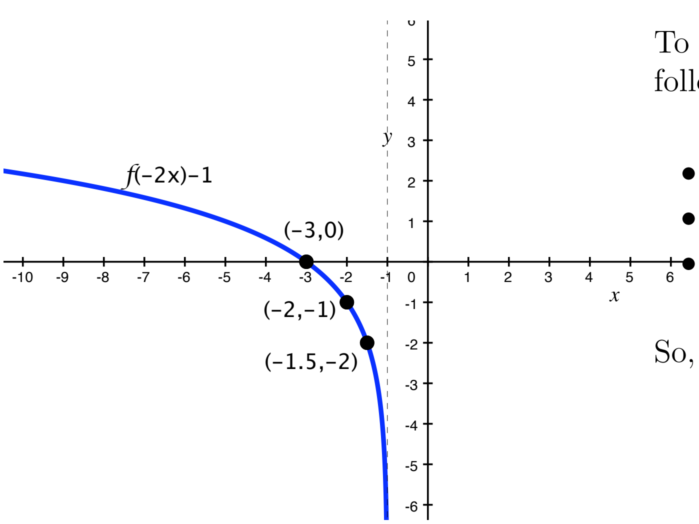
   (b)



To get the function $2f(x+3)$, we have the following transformations on $f(x)$:

- Horizontal shift left by 3
- Vertical stretch by 2

   (c)



To get the function $f(-2x) - 1$, we have the following transformations on $f(x)$:

- Horizontally compress $f(x)$ by $\frac{1}{2}$
- Reflect over the y-axis
- Shift down by 1

So, the domain is $\boxed{x < -1 \text{ or } (-\infty, -1).}$