Math 111
Exam 1 Solutions

1. (a) S is a function of t, so we can organize our data in points of the form (t, S).

$$(0,\ 30000) \quad \text{and} \quad (5,\ 34000)$$

The rate of change of the salary over time (slope) is given by

$$m = \frac{34000-30000}{5-0} = 800 \text{ dollars/year.}$$

Since the initial value of the salary is \$30000, the function is given by $\boxed{S = 800t + 30000.}$

(b) We want to find t when $S = 50000$. $\Rightarrow$ $50000 = 800t + 30000$

$$t = 25 \text{ years}$$

So, the salary is \$50,000 in the year $\boxed{2025.}$

2. (a) We cannot plug in values for x that would make the denominator equal to 0.

Since $x(x-1) = 0$ when $x = 0$ or $x = 1$, we have that the domain is
$\boxed{\text{all real numbers except } x = 0 \text{ and } x = 1}$ OR $\boxed{x \neq 0 \text{ and } x \neq 1.}$

(b) Note that $f(2) = \frac{12}{2(2-1)} = 6$ and $f(4) = \frac{12}{4(4-1)} = 1.$

So, the rate of change is $\frac{f(4)-f(2)}{4-2} = \frac{1-6}{4-2} = \boxed{-\frac{5}{2}.}$

3. (a) Plugging in $t = 16$: $D = 3(\sqrt[4]{16}) = 3(2) = \boxed{6 \text{ inches.}}$

(b) Solving the equation $12 = 3\sqrt[4]{t}$ $\Rightarrow$ $4 = \sqrt[4]{t}$ $\Rightarrow$ $t = 4^4 = \boxed{256 \text{ seconds}}$

(c) Solving for t: $D = 3(\sqrt[4]{t})$ $\Rightarrow$ $\frac{D}{3} = \sqrt[4]{t}$ $\Rightarrow$ $\left(\frac{D}{3}\right)^4 = t$

So, the inverse function is $\boxed{t = f^{-1}(D) = \left(\frac{D}{3}\right)^4.}$

4. (a) Note that $g(1) = 4(1)^2 - 4(1) - 1 = -1.$ So, $h(g(1)) = h(-1) = \sqrt{-1+10} = \sqrt{9} = \boxed{3.}$

(b) $h(g(x)) = h(4x^2 - 4x - 1) = \sqrt{4x^2 - 4x - 1 + 10} = \boxed{\sqrt{4x^2 - 4x + 9}}$

5. (a) Since the coefficient of x^2 is positive, $g(x)$ is a parabola that opens up, so it is $\boxed{\text{concave up.}}$

(b) If the x-coordinate of a point on the graph is 2, then $x = 2$. We can find the y-coordinate by plugging $x = 2$ into the function.

$$y = g(2) = 4(2)^2 - 4(2) - 1 = 7.$$

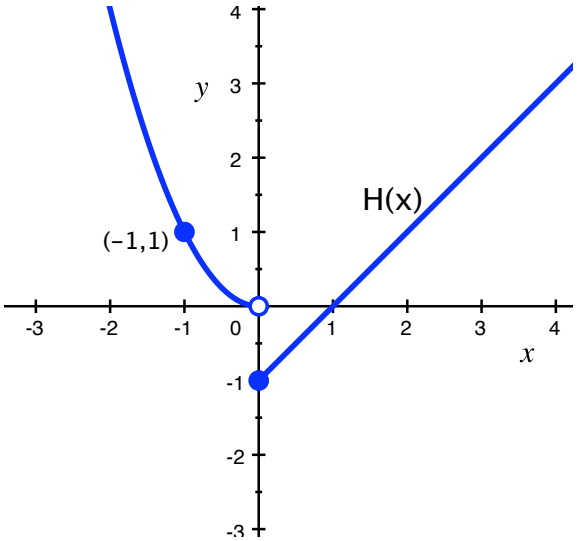
So, the only point with an x-coordinate of 2 on the graph is $\boxed{(2, 7).}$

(c) We must solve $4x^2 - 4x - 1 = 2 \quad \Rightarrow \quad 4x^2 - 4x - 3 = 0.$

This can be solved by factoring or by using the quadratic formula.
If factoring: $(2x + 1)(2x - 3) = 0 \quad \Rightarrow \quad \boxed{x = -\frac{1}{2}, \ x = \frac{3}{2}.}$

6. (a) For $x < 0$, the graph of $H(x)$ is given by $y = x^2$, which is a parabola. You can plot points to get a good sketch of its shape.

For $x \geq 0$, the graph of $H(x)$ is given by $y = x - 1$, which is a line with slope 1 and y-intercept $(0, -1)$.



(b) The function $h(x)$ will equal 4 if the functions $y = x - 1$ and $y = x^2$ are equal to 4 on their respective domains.

Note that $x - 1 = 4$ if $x = 5$ and $x^2 = 4$ if $x = \pm 2$. (For $x = \pm 2$, note that x^2 is only defined for $x < 0$, so we actually only get $x = -2$ as one of the solutions.)

So, $h(x) = 4$ if $\boxed{x = 5 \quad \text{and} \quad x = -2.}$

(c) The range is the y-values that the function attains $\Rightarrow$ The range is $\boxed{y \geq -1 \ \text{ or } \ [-1, \infty).}$