Math 111 Worksheet #8 Solutions

1. Solve the following equations.

   Note: You can use any logarithm to solve these exponential functions.

   (a) $2e^{-.1t} = 6$

   Dividing both sides by 2: $e^{-.1t} = 3$

   Taking the natural log of both sides:
   $$\begin{aligned} \ln(e^{-.1t}) &= \ln 3 \\ -.1t \cdot \ln e &= \ln 3 \\ -.1t &= \ln 3 \\ t &= \frac{\ln 3}{-.1} \approx -10.986 \end{aligned}$$

   Check: $2e^{-.1(-10.986)} \approx 6$ ✓

   (b) $49(2.75)^{x-1} = 11(3.5)^x$

   Taking the common log of both sides:
   $$\begin{aligned} \log(49(2.75)^{x-1}) &= \log(11(3.5)^x) \\ \log 49 + \log(2.75)^{x-1} &= \log 11 + \log(3.5)^x \\ \log 49 + (x-1)\cdot\log 2.75 &= \log 11 + x\cdot\log 3.5 \\ \log 49 + x\cdot\log 2.75 - \log 2.75 &= \log 11 + x\cdot\log 3.5 \end{aligned}$$

   Putting variable terms on one side of the equation:
   $$x\cdot\log 2.75 - x\cdot\log 3.5 = \log 11 + \log 2.75 - \log 49$$

   Factoring x from each term:
   $$x(\log 2.75 - \log 3.5) = \log 11 + \log 2.75 - \log 49$$

   $$\Rightarrow \quad x = \frac{\log 11 + \log 2.75 - \log 49}{\log 2.75 - \log 3.5} = 2$$

   Check: $49(2.75)^{2-1} = 134.75$
   $11(3.5)^2 = 134.75$ ✓

2. What is the doubling time for a savings account that gives 5% interest per year? (How long does it take a certain amount in the account to double?)

   For an account with 5% interest, the growth factor is 1.05.
   So, the account will have doubled when $2 = 1.05^t$ for some time t in years.

   Taking the common log of both sides:
   $$\begin{aligned} \log 2 &= \log(1.05^t) \\ \log 2 &= t\cdot\log 1.05 \\ \Rightarrow \quad t &= \frac{\log 2}{\log 1.05} \approx 14.207 \text{ years} \end{aligned}$$

   So, it will take approximately 14.2 years to double your money in an account paying 5% interest per year.

3. In order to double your money in 5 years, what yearly interest rate would an account need to have? (Note: You won't need logarithms to solve this equation.)

   In this problem, the interest rate is unknown. Suppose the growth factor is $(1 + r)$.

   Then we want to find r so that $\quad 2 = (1 + r)^5 \quad$ (Doubled after 5 years)

   Taking the fifth root of both sides: $\quad \sqrt[5]{2} = 1 + r$

   $$1.148698 \approx 1 + r$$

   $$\Rightarrow \quad r \approx .148698$$

   So, you would need an account with 14.8698% interest to double your money in 5 years.

4. Find the domain of the following functions.

   (a) $\ln(2 - x)$

   For natural log (or any log) to be defined, we must have that the inner expression be positive $\Rightarrow \quad 2 - x > 0 \quad \Rightarrow \quad 2 > x$

   So, the domain of $\ln(2 - x)$ is $\quad x < 2 \quad$ or $\quad (-\infty, 2)$.

   (b) $\ln(e^x)$

   There are a couple of ways to think about this function.

   - Again, for the natural log to be defined, we must have that the inner expression be positive $\quad \Rightarrow \quad e^x > 0$.

     However, note that e^x is positive for all values of x. So, the domain of $\ln(e^x)$ is all real numbers.

   - Note that $\ln(e^x) = x \cdot \ln e = x \quad$ for any number x.
     The domain of x is all real numbers, so the domain for $\ln(e^x)$ is all real numbers.