Math 111 Worksheet #6 Solutions

1. In the region of Western Hudson Bay of Canada, the polar bear population in 1987 was estimated to be 1200. In 2007, it was estimated to be 950. Let t be the number of years after 1987.

   (a) Assuming the population P is decreasing exponentially, find a formula giving P as a function of t.

   Note that when $t = 0$ (year 1987), we have that $P = 1200$ (initial population).

   So, given that $P = a \cdot b^t$, we know that $a = 1200 \quad \Rightarrow \quad P = 1200b^t$.

   To find b, we can plug in the values $t = 20$ (year 2007) and $P = 950$.

$$950 = 1200b^{20}$$

$$\frac{950}{1200} = b^{20}$$

$$\Rightarrow \quad b = \left(\frac{950}{1200}\right)^{1/20} \approx .988387$$

   Formula: $P = 1200(.988387)^t$

   (b) According to your formula, what is the percentage change in the population per year?

   Consider $1 + r = .988387 \quad \Rightarrow \quad r = .011613$.
   So, according to the model, the population is decreasing by approximately 1.1613% each year.

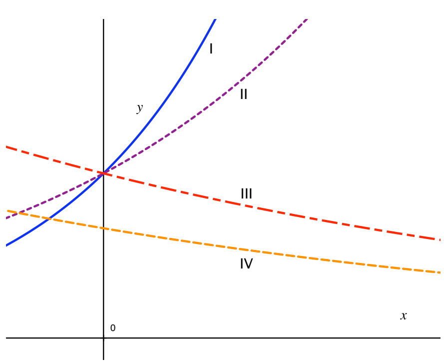
   (c) What does your formula predict for the population in 2017? In 2047? In 2087?

   Year 2017 Plugging $t = 30$ into the formula: $P = 1200(.988387)^{30}$
   ≈ 845 polar bears
   (Rounded to the nearest bear.)

   Year 2047 Plugging $t = 60$ into the formula: $P = 1200(.988387)^{60}$
   ≈ 595 polar bears
   (Rounded to the nearest bear.)

   Year 2087 Plugging $t = 100$ into the formula: $P = 1200(.988387)^{100}$
   ≈ 373 polar bears
   (Rounded to the nearest bear.)

2. Match the following graphs with the possible functions.



   (a) $3(1.25)^x$

   (b) $2(.9)^x$

   (c) $3(1.5)^x$

(d) $3(.9)^x$

Note that exponential functions have the form $a \cdot b^x$ for constants a and b.

Three of the graphs (I, II, III) have the same y-intercept and graph IV has a y-intercept with a smaller y-value. Looking at the possible functions, we must have that graph IV corresponds to $2(.9)^x$, since it has the smallest value of a.

Graph III is decreasing, so it must have a growth factor b that is less than 1. So, graph III must correspond to the function $3(.9)^x$.

Graphs I and II are increasing, so they both have a growth factor b that is greater than 1. Since graph I grows faster than graph II, the growth factor for graph I must be greater than the growth factor for graph II. Thus, graph I must be $3(1.5)^x$ and graph II must be $3(1.25)^x$.

3. Sketch a graph of $f(x) = -3^x$ for $-3 \leq x \leq 3$.