Math 111 Worksheet #5 Solutions

1. Account one starts with \$10000 and increases by 10% a day. Account two begins with one penny and doubles each day (grows by 100% a day).

   (a) Fill in the following table for the two functions. How much do you have in each account after 1 day? 2 days? 3 days?

   - Account One:

     Since the account increases by 10% a day, for each day, we must multiply by a factor of 1+.10=1.1 to get the account balance.

     After 1 day: $10000(1.1) = 11000$ dollars

     After 2 days: $11000(1.1) = 12100$ dollars
     (Note: This is the same as the following: $10000(1.1)(1.1) = 10000(1.1)^2$.)

     After 3 days: $12100(1.1) = 13310$ dollars
     (Note: This is the same as the following: $10000(1.1)(1.1)(1.1) = 10000(1.1)^3$.)

   - Account Two:

     Since the account increases by 100% a day, for each day, we must multiply by a factor of 1+1.00=2 to get the account balance (Doubles each day).

     After 1 day: $.01(2) = .02$ dollars

     After 2 days: $.02(2) = .04$ dollars
     (Note: This is the same as the following: $.01(2)(2) = .01(2)^2$.)

     After 3 days: $.04(2) = .08$ dollars
     (Note: This is the same as the following: $.01(2)(2)(2) = .01(2)^3$.)

   (b) Let A_1 = the amount in account one on day t and A_2 = the amount in account two on day t. Write formulas for A_1 and A_2 in terms of t.

   $$A_1 = 10000(1.1)^t \quad \text{and} \quad A_2 = .01(2)^t$$

   (c) How much does each account have on day 15? How much on day 25? How much does each account have after 31 days?

   - Account One:

     Day 15: $A_1 = 10000(1.1)^{15} \approx 10000(4.177248) = 41,772.48$ dollars

     Day 25: $A_1 = 10000(1.1)^{25} \approx 10000(10.834706) = 108,347.06$ dollars

     Day 31: $A_1 = 10000(1.1)^{31} \approx 10000(19.194343) = 191,943.43$ dollars

- Account Two:

  Day 15: $A_1 = .01(2)^{15} \approx .01(32768) = 327.68$ dollars

  Day 25: $A_1 = .01(2)^{25} \approx .01(33554432) = 335,544.32$ dollars

  Day 31: $A_1 = .01(2)^{31} \approx .01(2147483648) = 21,474,836.48$ dollars

Note: For an investment in the long-term, a faster rate of growth is better than a larger initial invested amount.

2. (a) Find the point on the graph of $y = 2x^2 - 2x - 24$ whose x-coordinate is 1.

   We need to find the point (x, y) whose x-coordinate is 1 $\Rightarrow$ $(1, y)$.
   So, given $x = 1$, we must find y.
   Using the equation, $y = 2(1)^2 - 2(1) - 24 = -24$.
   $\Rightarrow$ Point: $(1, -24)$

   (b) Find the zeros of $y = 2x^2 - 2x - 24$.

   The zeros of the equation are x-values such that $2x^2 - 2x - 24 = 0$.

   Factoring the left-side of the equation: $2(x^2 - x - 12) = 0$
   $2(x - 4)(x + 3) = 0$

   So, $x = 4$ or $x = -3$.
   (Note: The constant 2 can never equal zero, so we only need to know when $x - 4$ and $x + 3$ are equal to zero.)

3. Suppose the population of town A (in thousands) is given by $P_A = 24(1.08)^t$ and the population of town B (in thousands) is given by $P_B = 35(.78)^t$ for year t after 2000.

   (a) What is the population of each town in 2000?

   The population of town A in the year 2000 can be found by plugging in $t = 0$ into the population function. $P_A(0) = 24(1.08)^0 = 24(1) = 24$ thousand people.

   The population of town B in the year 2000 can be found by plugging in $t = 0$ into the population function. $P_B(0) = 35(.78)^0 = 35(1) = 35$ thousand people.

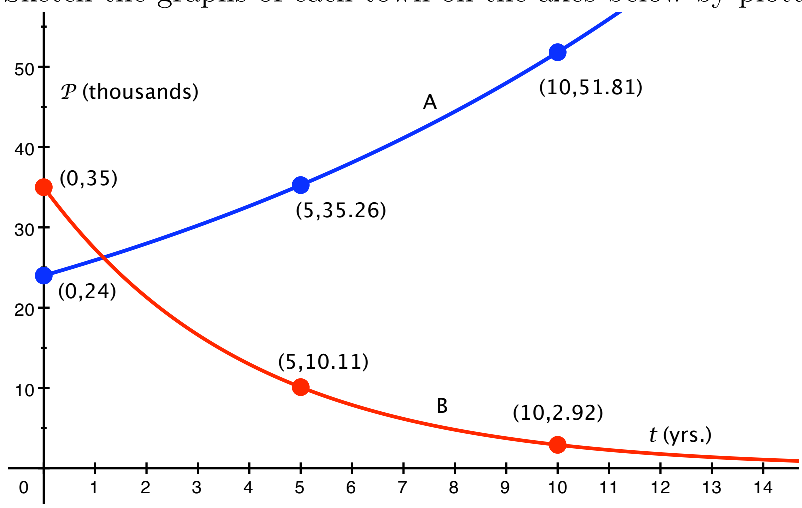
   (Note: The initial population of each town is given by the number that is multiplying the term with an exponent in each function.)

   (b) What is the growth rate of each town?

   Town A has a growth rate of .08 $\Rightarrow$ It is growing by 8% a year.

   Town B has a growth rate of $-.22$ $\Rightarrow$ It is shrinking by 22% a year.

(c) Sketch the graphs of each town on the axes below by plotting at least 3 points.



I have chosen to plot the values at $t = 0$, $t = 5$, and $t = 10$. Any 3 points for each graph will work as well. You should see the same shapes for each graph as above.