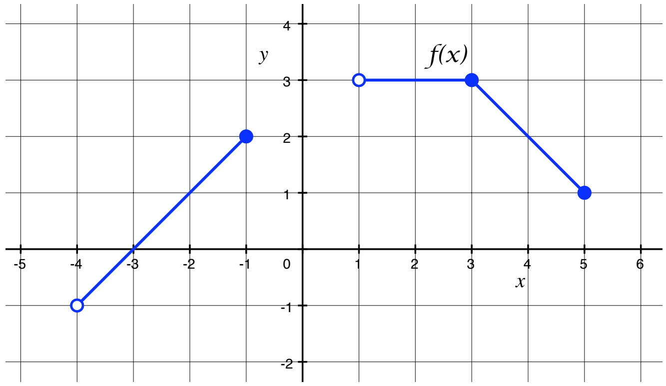
Math 111 Worksheet #3 Solutions

1. The following is a graph of the piecewise-defined function $f(x)$.



Note: Each point on the graph of f is of the form $(x, y) = (x, f(x))$.*

(a) Evaluate $3f(4) - f(2)$.

Note that $(4, 2)$ is a point on the graph of f. This means that when $x = 4$, $y = f(4) = 2$.

Similarly, the point $(2, 3)$ is on the graph, which means that when $x = 2$, $y = f(2) = 3$.

So, $3f(4) - f(2) = 3(2) - 3 = 3$.

(b) Find all x such that $f(x) = 1$.

Looking at the graph, we can see that the only points with y-coordinate of 1 are $(-2, 1)$ and $(5, 1)$. So, when $x = -2$ or $x = 5$, $f(x) = 1$.

(c) Solve $f(x) = 2$.

This problem is similar to part (b). We want to find values of x for which $f(x) = 2$.

Looking at the graph, we can see that the only points with y-coordinate of 2 are $(-1, 2)$ and $(4, 2)$. So, when $x = -1$ or $x = 4$, $f(x) = 2$.

(d) State the domain and range of f.

The domain of f is $-4 < x \leq -1$ and $1 < x \leq 5$.
(In interval notation: $(-1, -1] \cup (1, 5]$)

The range of f is $-1 < y \leq 3$.
(In interval notation: $(-1, 3]$)

(e) Write a formula for $f(x)$.

On the interval $-4 < x \leq -1$, we have a line of slope 1. The y-intercept will be $(0, 3)$, which will give us the linear equation $y = x + 3$. To find the equation, you could also use the fact that the line goes through the point $(-1, 2)$ and use the point-slope form of the line $\Rightarrow \quad y - 2 = 1(x - (-1)) \quad \Rightarrow \quad y = x + 3$.

On the interval $-1 < x < 3$ or $-1 < x \leq 3$, we have the horizontal line $y = 3$.

On the interval $3 \leq x \leq 5$ or $3 < x \leq 5$, we have a line of slope -1. The y-intercept will be $(0, 6)$, which will give us the linear equation $y = -x + 6$. You could also use the fact that the line goes through the point $(5, 1)$ and use the point-slope form of the line $\Rightarrow \quad y - 1 = -1(x - 5) \quad \Rightarrow \quad y = -x + 6$.

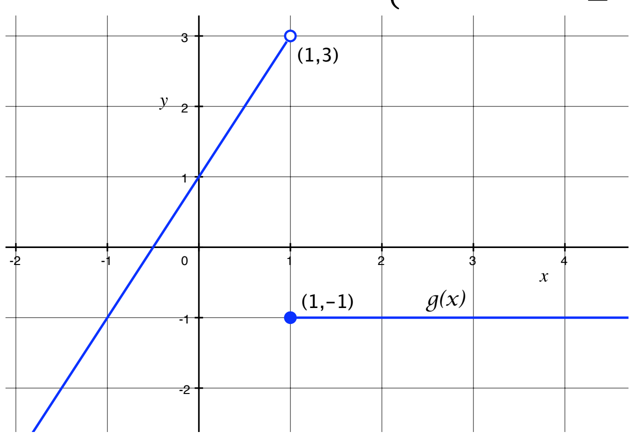
So, we have that $f(x)$ can be defined in the following ways:

$$f(x) = \begin{cases} x + 3 & \text{if } -4 < x \leq -1 \\ 3 & \text{if } 1 < x < 3 \\ -x + 6 & \text{if } 3 \leq x \leq 5 \end{cases}$$

OR $$f(x) = \begin{cases} x + 3 & \text{if } -4 < x \leq -1 \\ 3 & \text{if } 1 < x \leq 3 \\ -x + 6 & \text{if } 3 < x \leq 5 \end{cases}$$

(The difference is where $x = 3$ is included in the domain.)

2. Sketch the function $g(x) = \begin{cases} 2x + 1 & \text{if } x < 1 \\ -1 & \text{if } x \geq 1 \end{cases}$. What is the range of the function?



The range will be $y < 3$ or $(-\infty, 3)$.

3. What is the domain of the function $h(x) = \frac{5}{x-2} + 3$?

Note that the only number that we could not plug into the function as an x-value is $x = 2$. (Avoid division by zero!)

So, the domain of the function is all real numbers except $x = 2$.