Math 111 Worksheet #13 Solutions

1. Find the vertex and line of symmetry of the quadratic function $f(x) = 2x^2 - 12x + 10$. Is the graph of $f(x)$ opening up or down?

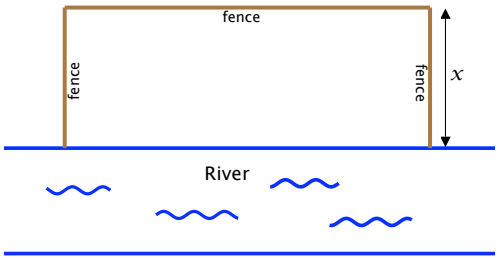
   Using the vertex formula:

   $$x\text{-coordinate of vertex:} \quad x = -\frac{-b}{2a} = -\frac{-12}{2(2)} = 3$$
   $$y\text{-coordinate of vertex:} \quad y = f(3) = -8$$

   $$\Rightarrow \quad \text{Vertex: } (3,-8), \qquad \text{Line of Symmetry: } x = 3$$

   Since the coefficient of x^2 is positive ($a = 2$), the graph of $f(x)$ is a parabola that **opens up**.

2. A farmer wants to enclose a rectangular area against a straight riverbank with 3 sides of fencing. He can afford 100 feet of fencing. Let x be the width of the enclosure. (See figure below.)



   (a) Find a formula that gives the area of the rectangular enclosure given the width x.

   Since the farmer has 100 feet of fencing and the width of the rectangle is x feet, the length of the rectangle must be $100 - 2x$ feet. (Length $= 100 - x - x = 100 - 2x$)

   So, the area of the rectangle is given by
   $$\begin{aligned} \text{Area} &= \text{Length} \times \text{Width} \\ &= (100 - 2x)x \\ &= 100x - 2x^2 \\ &= -2x^2 + 100x \quad \text{(Standard Form)} \end{aligned}$$

   (b) What are the dimensions of the enclosure with maximum area? What is the maximum area?

   Note that our equation for the area of the rectangular enclosure is a quadratic function. It has a maximum since the graph is a parabola that opens down ($a < 0$).

   Using the vertex formula:
   $$x = -\frac{-b}{2a} = -\frac{100}{2(-2)} = 25$$

   This means that the area of the rectangle is maximum when the width of the rectangle is 25 feet. When the width of the rectangle is 25 feet, the length is 50 feet, so the dimensions

that give the maximum area are 25 feet by 50 feet.

The maximum area is $25 \times 50 = 1250$ ft^2.

3. Decompose the function $h(x) = \sqrt{2e^x + 1}$ into 2 new functions $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$. (Note: Answers may vary.)

   Here are some possible answers.

   - $f(x) = \sqrt{x}, \quad g(x) = 2e^x + 1 \quad \Rightarrow \quad f(g(x)) = \sqrt{2e^x + 1}$
   - $f(x) = \sqrt{2x + 1}, \quad g(x) = e^x \quad \Rightarrow \quad f(g(x)) = \sqrt{2e^x + 1}$