

**Math 111**  
**Exam 2 Solutions**

1. (20 pts.) Suppose you have an account that gives interest at 9% compounded quarterly.
- (a) (12 pts.) How much would you need to invest in the account today to have \$3000 in the account 8 years from now?

Let  $P$  be the amount that you need to deposit today.

$$\begin{aligned} \text{Then } 3000 &= P\left(1 + \frac{.09}{4}\right)^{4(8)} &\Rightarrow & 3000 = P(1.0225)^{32} \\ & &\Rightarrow & P = \frac{3000}{1.0225^{32}} = \$1471.96 \end{aligned}$$

- (b) (8 pts.) What is the effective rate of the account?

The effective rate is given by  $\left(1 + \frac{.09}{4}\right)^4 - 1 = 1.0225^4 - 1 \approx 0.09308$  or 9.308%.

2. (8 pts.) If I borrow \$15000 at 6% simple interest for 18 months, how much would I owe at the end of the loan?

Let  $A$  be the amount that I owe at the end of the loan. Note that 18 months is 1.5 years.

$$\text{Then } A = 15000(1 + .06(1.5)) = 15000(1.09) = \$16350.$$

3. (25 pts.) A population of flying monkeys is given by  $P(t) = 5e^{0.5t}$  in **thousands** at year  $t$ .
- (a) (7 pts.) What is the population after 3 years? (Approximate to the nearest monkey.)

The population after 3 years is given by  $P(3) = 5e^{5(3)} \approx 22.40844535$  thousand monkeys.

Rounding to the nearest monkey:

$$\begin{aligned} 22.40844535 \text{ thousand monkeys} &= 22408.44535 \text{ monkeys} \\ &\approx 22408 \text{ monkeys} \end{aligned}$$

- (b) (12 pts.) When is the population equal to 20,000 flying monkeys?

We want to find  $t$  such that  $P(t) = 20$  since  $P$  is in thousands.

$$\begin{aligned} \Rightarrow 5e^{.5t} &= 20 \\ e^{.5t} &= 4 \\ \ln(e^{.5t}) &= \ln 4 \quad (\text{Common log can also be used.}) \\ .5t &= \ln 4 \\ t &= 2 \ln 4 \approx 2.7726 \text{ years} \end{aligned}$$

- (c) (6 pts.) What is the percentage change in the population each year? Is the population increasing or decreasing?

Note that  $e^{.5} \approx 1.6487$ . So, the percentage change in the population each year is approximately 64.87% and the population is increasing.

4. (20 pts.) Solve the following equations.

(a) (12 pts.)  $16(0.2)^{-x} = 20^x$

Taking (common or natural) log of both sides:

$$\log(16(0.2^{-x})) = \log(20^x)$$

$$\log 16 + \log(0.2^{-x}) = x \cdot \log 20$$

$$\log 16 - x \cdot \log 0.2 = x \cdot \log 20$$

$$\log 16 = x \cdot \log 0.2 + x \cdot \log 20$$

$$\log 16 = x(\log 0.2 + \log 20)$$

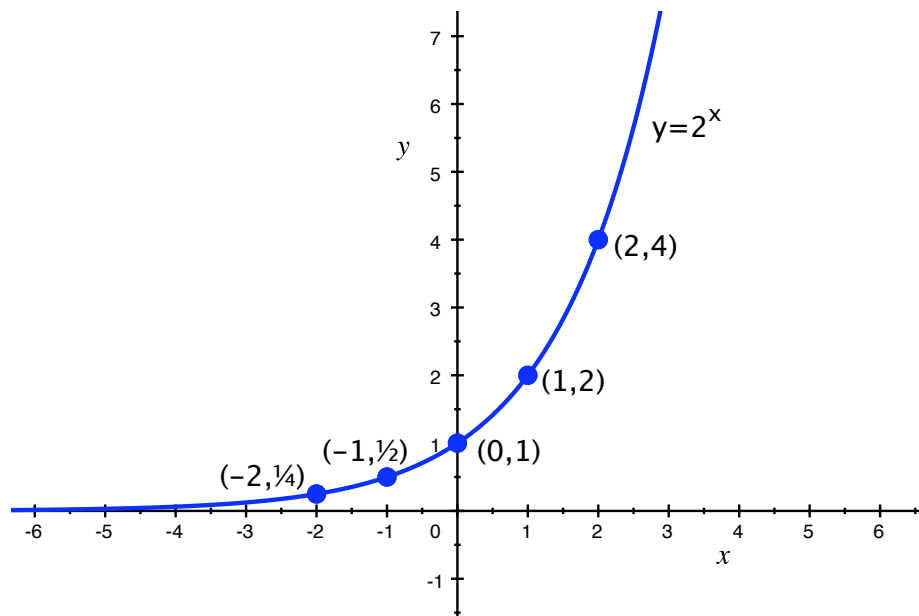
$$\Rightarrow x = \frac{\log 16}{\log 0.2 + \log 20} = 2$$

(b) (8 pts.)  $\log(4x + 8) = 2$

By the definition of the common log, we have that  $4x + 8 = 10^2 \Rightarrow x = 23$ .

5. (12 pts.)

- (a) (8 pts.) Sketch a graph of  $h(x) = 2^x$  on the axis below. Plot and label at least 3 points.



- (b) (4 pts.) What is the equation of the asymptote of  $h(x) = 2^x$ ?  
The function  $h(x)$  has the horizontal asymptote  $y = 0$ .