# Math 111
# Exam 1 Solutions

1. You sell "I ♡ Linear Functions" t-shirts and you begin to keep track of your daily sales. You notice that you sell 420 shirts if the price for each shirt is \$8. You sell 308 t-shirts if the price is \$12 each. Let $p =$ the price of each shirt and $q =$ the quantity sold in one day.

   (a) Find a formula for $q$ in terms of $p$ assuming that $q$ is a **linear** function of $p$, i.e., $q = f(p)$.

   Points of the function $q = f(p)$ will have the form $(p, q)$. So, we have the points $(8, 420)$ and $(12, 308)$ to use to write the linear function.

   Finding the rate of change of $f$ = slope: $m = \frac{308-420}{12-8} = \frac{-112}{4} = -28$ shirts/dollar

   To write the equation, we can use either the point-slope form or the slope-intercept form.

   - Point-Slope Form: Using the point $(8, 420)$, we have
     $$q - 420 = -28(p - 8)$$
     This can be simplified into $q = f(p) = -28p + 644$.

   - Slope-Intercept Form: Since the rate of change of $f$ is $-28$, we have
     $$q = -28p + b \quad \text{for some constant } b.$$
     Plugging in the values $p = 8$ and $q = 420$: $420 = -28(8) + b \quad \Rightarrow \quad b = 644$

     So, $q = f(p) - 28p + 644$.

   (b) How many shirts will you sell if the price is \$15?

   Using the formula from part (a), $q = -28(15) + 644 = 224$. So, at a price of \$15, we will sell 224 shirts.

   (c) Solve $f(p) = 490$ and interpret this in terms of $p$ and $q$.

   To solve $f(p) = 490$: $-28p + 644 = 490 \quad -28p = -154$
   $$\Rightarrow \quad p = 5.5$$

   Interpretation: If we sell the shirts for \$5.50, then 490 shirts will be sold.

   (d) Find the inverse function $p = f^{-1}(q)$.

   Solving for the variable $p$: $q - 644 = -28p \quad \Rightarrow \quad p = \frac{q-644}{-28}$

   So, $p = f^{-1}(q) = \frac{q-644}{-28}$.

2. Consider the two functions $f(x) = 3x + 1$ and $g(x) = -x^3 + 5$.

   (a) Find $f(g(2))$.

   Note that $g(2) = -2^3 + 5 = -8 + 5 = -3$.

   So, $f(g(2)) = f(-3) = 3(-3) + 1 = -9 + 1 = -8$.

   (b) Find $f(g(x))$. Simplify as much as possible.

   $f(g(x)) = f(-x^3 + 5) = 3(-x^3 + 5) + 1 = -3x^3 + 15 + 1 = -3x^3 + 16$

3. (a) What is the domain of the function $g(x) = \sqrt{3x + 12}$?

   In order for the square root function to be defined, we must have that $3x + 12 \geq 0$

   $$\Rightarrow \quad 3x \geq -12$$
   $$\Rightarrow \quad x \geq -4$$

   The domain is $x \geq -4$ or $[-4, \infty)$.

   (b) What is the range of the function $h(x) = 3x - 2$ with domain $-2 \leq x < 6$? (Hint: Consider the graph of $h$.)

   Since $h(x)$ is a linear function that has positive slope, it is rising from left to right. This means that the smallest $y$-value that the function attains is $h(-2) = -8$, the largest $y$-value is $h(6) = 16$, and $h(x)$ will attain all the $y$-values in between $-8$ and 16.

   So, the range is $-8 \leq h(x) \leq 16$ or $[-8, 16]$.

4. Write an equation of a line that intersects the $x$-axis at $x = 3$ and is perpendicular to the line $y = \frac{1}{3}x + 4$.

   The line $y = \frac{1}{3}x + 4$ has slope $\frac{1}{3}$, so the line that we are writing an equation for is $-3$ (negative reciprocal of $\frac{1}{3}$).

   Since it intersects the $x$-axis at $x = 3$, the line goes through the point $(3, 0)$.

   Using the point-slope form of a linear equation: $y - 0 = -3(x - 3) \quad \Rightarrow \quad y = -3x + 9$

5. Consider the function $f(x) = 3x^2 - 2x - 3$.

   (a) Find all of the points on the graph of $f$ whose $y$-coordinates are 5.

We are trying to find values of $x$ so that $f(x) = 5$ $\Rightarrow$

$$\begin{aligned} 3x^2 - 2x - 3 &= 5 \\ 3x^2 - 2x - 8 &= 0 \\ (3x+4)(x-2) &= 0 \end{aligned}$$

So, $x = -\frac{4}{3}$ or $x = 2$. (You can also use the quadratic formula to find $x$.)

So, the points on the graph of $f$ with $y$-coordinate of 5 are $(-\frac{4}{3}, 5)$ and $(2, 5)$.

(b) Find the rate of change of $f$ between $x = 0$ and $x = 2$.

Rate of Change of $f$ between $x = 0$ and $x = 2$: $\frac{f(2)-f(0)}{2-0} = \frac{5-(-3)}{2} = \frac{8}{2} = 4$