Math 126 The Chain Rule

1. If R is the total resistance of two resistors, connected in parallel, with resistances R_1 and R_2, then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Suppose $R_1 = 30\ \Omega$ and $R_2 = 60\ \Omega$ and that both R_1 and R_2 are increasing at the rate of $\frac{1}{2}\ \Omega$/sec. Find the rate at which the total resistance is changing.

2. If the number of moles of an ideal gas is kept constant, then the pressure, volume, and temperature of the gas are related by the equation $PV = kT$, where k is a constant. Suppose the pressure of a gas is one atmosphere and that the pressure is increasing at the rate of 0.1 atm/sec. Suppose the volume is two liters and that the volume is decreasing at the rate of 0.3 liters/sec. If the temperature is 10 K at this time, find the rate at which the temperature is changing.

3. The temperature at the point (x, y) is given by the function $T(x, y) = \sqrt{x^2 + y^2}$. The position at time t seconds of a particle P moving in the plane is given by the function

$$\vec{r}(t) =< t^6 - 2t^5 + 4, \quad t^5 - t^4 + 4t >$$

a. Find the rate at which the temperature of the particle's environment is changing at time $t = 1$ second.
b. Suppose that a second particle R moved along the same path as P in the same direction, but with twice the speed. Find the rate at which the temperature of R's environment is changing when it passes the point $(3, 4)$.

4. The temperature at the point (x, y) in the plane is given by the function $T(x, y) = xy^2 - y^3$. A particle is moving in the plane along the parabola $y = x^2$ at a constant speed. At the instant the particle passes the point $(1, 1)$ the temperature of its surrounding environment is increasing at the rate of 3 deg/sec. Find the speed of the particle.

5. A particle moves with a constant velocity on the plane $4x + 6y + z = 20$. The speed of the particle is 3 units/sec and the particle gains altitude at the rate of 2 units/sec. Find the velocity of the particle.