Math 126 Quiz #2 Name: ______________________________

Directions: Please show all your work to receive full credit. Give brief explanations for word problems and end with a concluding sentence. Define any variables you introduce.

1. (6 points) Find all vectors of length 2 orthogonal to both $\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{2j} + \mathbf{k}$.

2. (6 points) Find an equation of the line through the points $(2, 1, 4)$ and $(1, -2, 3)$.

3. (6 points) Find an equation of the plane through $(2, 1, 0)$ and perpendicular to the line $\mathbf{r} = < 2, 6, 2 > + t < 3, 2, -1 >$.

4. (7 points) Find the distance between the skew lines

$$\mathbf{r}_1 = < 1, 1, 0 > + t < 1, 4, 2 >$$

and

$$\mathbf{r}_2 = < 1, 5, -2 > + t < 2, 1, -3 >.$$

$\sqrt{x^2+y^2+z^2} = 2$
$x^2+y^2+z^2 = 4$

[1.] Find all vector of length 2 orthogonal to both $\vec{a} = \langle 1, 1, -1 \rangle$, $\vec{b} = \langle 0, 2, 1 \rangle$

sphere?

$$\vec{a} \times \vec{b} = \begin{vmatrix} 1, & 1, & -1 \\ 0, & 2, & 1 \end{vmatrix} = \langle 1+2, -(1), 2 \rangle$$

$$= \langle 3, -1, 2 \rangle \Rightarrow \text{direction.}$$

$$\frac{\langle 3, -1, 2 \rangle}{\sqrt{9+1+4}} = \frac{1}{\sqrt{14}} \langle 3, -1, 2 \rangle \Rightarrow$$ It has lengh of 1 and $\langle 3, -1, 2 \rangle$ direction.

$\therefore$ Length 2 $= \frac{2}{\sqrt{14}} \langle 3, -1, 2 \rangle \Rightarrow$ answer

or $-\frac{2}{\sqrt{14}} \langle 3, -1, 2 \rangle$

[2] $A(2, 1, 4)$ and $B(1, -2, 3)$
Find line. thru the point A, B

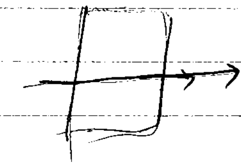
sol) $\vec{AB} = \langle -1, -3, -1 \rangle \Rightarrow$ direction of line.
$C = (x, y, z) \Rightarrow$ Point on the line.

$\therefore \vec{AC} = t\vec{AB}$

$\therefore \langle x-2, y-1, z-4 \rangle = t \langle -1, -3, -1 \rangle$

$$\therefore \left.\begin{aligned} x &= -t+2 \\ y &= -3t+1 \\ z &= -t+4 \end{aligned}\right\} \Rightarrow \text{answer.}$$

3 throug $A(2,1,0)$ and perpendicular $r = \langle 2,6,2\rangle + t\langle 3,2,-1\rangle$



Find eq of Plane

$\therefore$ The Plane has $\vec{n} = \langle 3,2,-1\rangle$ ✓

$B = (x,y,z) \rightarrow$ Point on the plane ✓

$\therefore \vec{AB} \cdot \vec{n} = 0$

$\langle x-2, y-1, z\rangle \cdot \langle 3,2,-1\rangle = 0$ ✓

$\therefore 3(x-2) + 2(y-1) - z = 0$

$\therefore 3x - 6 + 2y - 2 - z = 0$ ✓

$3x + 2y - z = 8$

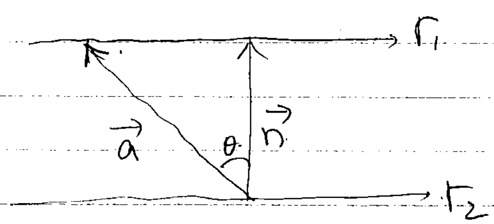
$\therefore$ Equation of plane

$= 3x + 2y - z = 8$

4 Find the distance between the skew lines

$r_1 = \langle 1,1,0\rangle + t\langle 1,4,2\rangle \rightarrow \vec{v} = \langle 1,4,2\rangle \Rightarrow$ direction

$r_2 = \langle 1,5,-2\rangle + t\langle 2,1,-3\rangle \quad \vec{u} = \langle 2,1,-3\rangle \Rightarrow$ direction



✓

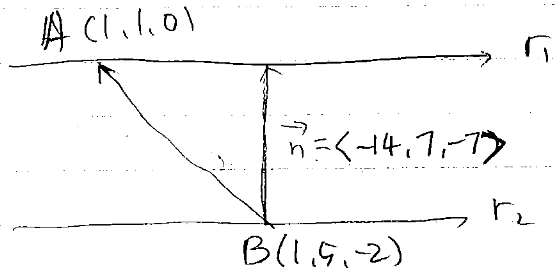
To get $\vec{n}$ which orthogonal both r_1 and r_2

$$\vec{v} \times \vec{u} = \begin{vmatrix} 1, 4, 2 \\ 2, 1, -3 \end{vmatrix} = \langle -12-2, -(-3-4), 1-8\rangle$$

$$= \langle -14, 7, -7\rangle$$ ✓

Choose any point on r_1 and r_2.

$A = (1, 1, 0)$ $\quad\quad$ $B = (1, 5, -2)$



$\overrightarrow{AB} = \langle 0, 4, -2 \rangle$ $\quad\quad$ $\overrightarrow{AB} \cdot \vec{n} = |AB||\vec{n}| \cos\theta$

$$\therefore d = |\overrightarrow{AB}| \cos\theta$$

$$= \frac{\overrightarrow{AB} \cdot \vec{n}}{\cancel{|\overrightarrow{AB}|}\,|\vec{n}|} \cancel{|\overrightarrow{AB}|} = \frac{|\overrightarrow{AB} \cdot \vec{n}|}{|\vec{n}|} = \mathrm{Comp}_{\vec{n}} \overrightarrow{AB}$$

$$\therefore d = \frac{|28 + 14|}{\sqrt{14^2 + 7^2 + 7^2}} = \cancel{2.45}\ \sqrt{6} \sim 2.45$$

$$\therefore d = 2.45$$