Math 163 Test #2 Name: ____________________

Directions: Please show all your work to receive credit. Give brief explanations for all problems and end with a concluding sentence. Remember that you will be graded on the clarity and organization of your work as well as its accuracy. **Do all your work on another sheet of paper. Calculators are not permitted.**

1. (10 points) Find the domain and range of the function

$$w = f(x, y, z) = \sqrt{9 - x^2 - y^2 - z^2}.$$

Sketch the domain.

2. (10 points) Find an equation of the tangent plane to the surface $1 + x - yz = 4\ln x + \ln xy + z^2$ at the point (1,1,1).

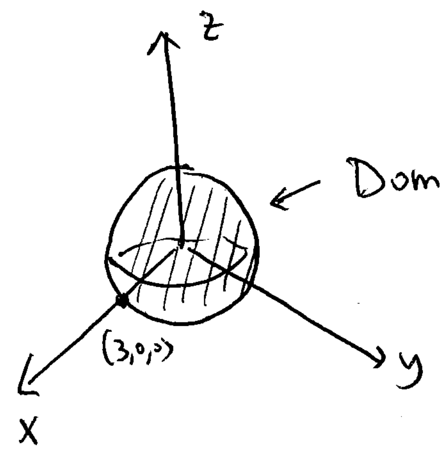
3. (10 points) The radius of a cylindrical can is increased by $p\%$. By approximately what percent must the height be decreased by in order that the volume of the can remain the same? Use the appropriate linear approximation. Assume that p is small.

4. (10 points) Near a buoy (located at the origin), the depth of a lake (in meters) at the point (x, y) is given by the function $f(x, y) = 200 + 0.02x^2 - 0.01y^2$, where x and y are measured in meters. A fisherman in a small boat is moving directly toward the origin at a speed of 5 m/sec. At what rate (with respect to time) is the depth of the water beneath the boat changing when the fisherman is located at the point $(50, 100)$?

5. (10 points) An ant is climbing a hill whose shape is given by the equation $z = 1000 - 0.01x^2 - 0.02y^2$. If the ant climbs the surface in the direction of steepest ascent, at what angle above the horizontal (ie. the xy-plane) will the ant be climbing when it passes the point $(60, 100, 764)$?

1. $D_f = \{ (x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 \leq 9 \}$

$R_f = \{ w \in \mathbb{R} \mid 0 \leq w \leq 3 \}$



Domain of f = Ball of radius 3 centered at (0,0,0).

2. Let $f(x,y,z) = 4\ln x + \ln(xy) + z^2 - x + yz$

Since our surface is a level surface of f (i.e. level 1), $\nabla f(1,1,1)$ is normal to the surface @ (1,1,1).

Since $\nabla f(x,y,z) = \langle \frac{4}{x} + \frac{1}{x} - 1, \frac{1}{y} + z, 2z + y \rangle$,

$\nabla f(1,1,1) = \langle 4, 2, 3 \rangle$.

An eqn. of the tangent plane to the surface @ (1,1,1) is

$4(x-1) + 2(y-1) + 3(z-1) = 0$.

3. Let V be the volume of a cylinder of radius r and height h. Then

$$V = \pi r^2 h$$

and

$$\Delta V \approx \frac{\partial V}{\partial r} \cdot \Delta r + \frac{\partial V}{\partial h} \cdot \Delta h = 2\pi r h \Delta r + \pi r^2 \Delta h$$

If $\Delta V = 0$, then

$$0 \approx 2\pi r h \,\Delta r + \pi r^2 \Delta h \,, \quad \text{so}$$

$$0 \approx \frac{2\pi r h}{\pi r^2 h} \Delta r + \frac{\pi r^2}{\pi r^2 h} \cdot \Delta h \,, \quad \text{or}$$

$$0 \approx 2\left(\frac{\Delta r}{r}\right) + \frac{\Delta h}{h} \,, \quad \text{so}$$

$$\frac{\Delta h}{h} \approx -2\left(\frac{\Delta r}{r}\right) = -2p\,\% .$$

Thus, the height should be decreased by about $2p\,\%$ if the volume is to remain constant.

4. Let $z = f(x,y)$ denote the depth of the water in meters and let t denote time in seconds.

The velocity of the fisherman when he is at the point $(50,100)$ is given by

$$\vec{V} = 5\left(\frac{\langle -1,-2\rangle}{|\langle -1,-2\rangle|}\right) = -\sqrt{5}\langle 1,2\rangle.$$

Since

$$\frac{dz}{dt} = (\nabla f)\cdot\vec{V},$$

$$\left.\frac{dz}{dt}\right|_{(x,y)=(50,100)} = \langle 0.04x, -0.02y\rangle\Big|_{(50,100)}\cdot\left(-\sqrt{5}\langle 1,2\rangle\right)$$

$$= -\sqrt{5}\langle 2,-2\rangle\cdot\langle 1,2\rangle$$

$$= 2\sqrt{5}$$

So the depth of the water beneath the boat is increasing at the rate of $2\sqrt{5}$ m/s when the fisherman is at the point $(50,100)$.

5. If $z = f(x,y) = 1000 - 0.01x^2 - 0.02y^2$, then

$$\nabla f(x,y) = \langle -0.02x, -0.04y \rangle$$

and $\nabla f(60,100) = \langle -1.2, -4 \rangle$.

Since $|\nabla f(60,100)| = \sqrt{17.44}$,

the angle that the path of the ant makes with the xy-plane when the ant is at the point $(60, 100, 764)$ is given by

$$\tan^{-1}(\sqrt{17.44}).$$