

Note: For some of these problems you will need Torricelli's law. One form of this law states that if a tank is filled with water and a small hole is punched in the bottom, then the rate (in say cm^3/s) at which the water drains from the tank is directly proportional to the square root of the depth of the water.

1. A container of arbitrary shape and no top is filled with water. If the rate of evaporation of the water in the tank is directly proportional to the area of the surface of the water, prove that the water level in the container drops at a constant rate.
- 2a. A cylindrical tank has a small hole in the bottom. The tank is filled with water and allowed to drain. Compare the rate at which the water level in the tank is dropping when the depth of the water is 40 feet to the rate at which the water level is dropping when the depth of the water is 10 feet.
- b. The parabola $y = x^2$ is rotated about the y -axis to form a bowl. A small hole is punched in the bottom of the bowl. The bowl is filled with water and allowed to drain. Compare the rate at which the water level in the tank is dropping when the depth of the water is 40 feet to the rate at which the water level is dropping when the depth of the water is 10 feet.
3. The curve $y = f(x)$ passes through the origin. The curve is rotated about the y -axis to form a bowl. A small hole is punched in the bottom of the bowl. The bowl is filled with water and allowed to drain. Determine all possible functions $f(x)$ so that the water level drops at a constant rate.
4. The parabola $y = x^2$ is rotated about the y -axis to form a container, where the units on the axes are measured in cm . At time $t = 0$ seconds the bowl is empty. Water is then poured into the container at a constant rate of $10 \text{ cm}^3/s$. At the same time a cylindrical container of radius 10 cm is being filled with water at the same rate. When is the water level in the two containers rising at the same rate?
5. The parabola $y = x^2$ is rotated about the y -axis to form a container, where the units on the axes are measured in cm . At time $t = 0$ seconds the bowl is empty. Water is then poured into the container at a constant rate of $10 \text{ cm}^3/s$. A bug crawls up the side of the container so that it remains just ahead of the advancing water. Find the speed of the bug at time $t = 5$ seconds.