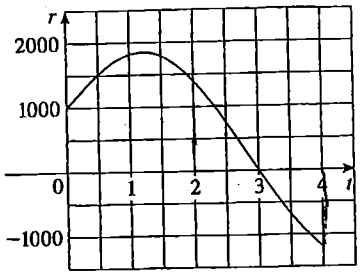
KEY

Math 152 Quiz #2

Directions: Please show all your work in an organized fashion to receive credit. Do all your work below the problems and on the back of this paper.

1. Water flows into and out of a storage tank. A graph of the net rate (in gal/day) at which water flows into the tank at time t days is given by the function $W(t)$, whose graph is shown below. Let $f(t) = \int_2^t W(x)\,dx$.



a. (4 points) Approximate $f(4)$. Explain its meaning. Include units.

b. (4 points) Approximate $f'(4)$. Explain its meaning. Include units.

c. (5 points) Sketch a graph of the function $f(t)$. Be sure to label the axes with units.

2. (12 points) Evaluate each of the following definite integrals.

a. $\int_0^1 (x^4 - 1)^2\,dx$

b. $\int_1^9 \frac{1}{2x}\,dx$

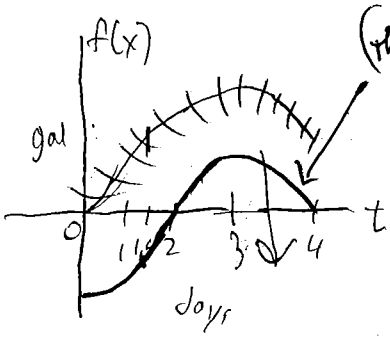
c. $\int_1^2 \frac{4+x^2}{x}\,dx$

d. $\int_0^1 \frac{e^{3x}+1}{e^x}\,dx$

1a) $f(4) \approx 0$ gal. This means that the net number of gallons that have flowed into the tank between day 2 and day 4 is approximately 0.

b) $f'(4) \approx -1200$ gal/day. This means that at $t = 4$ days, water is flowing out of the tank at approximately 1200 gallons per day. ✓

c) (this is correct)



maximum @ $t = 3$ days, minimum @ $t = 0$

inflection point @ $t = 1.5$ days

2) a) $\int_0^1 (x^4-1)^2\,dx = \int_0^1 (x^8-2x^4+1)\,dx = \int_0^1 x^8\,dx - \int_0^1 2x^4\,dx + \int_0^1 1\,dx$

$= \left[\frac{1}{9}x^9\right]_0^1 - \left[\frac{2x^5}{5}\right]_0^1 + [1]_0^1 = \frac{1}{9} - \frac{2}{5} + 1$ ✓

$\frac{5}{45} - \frac{18}{45} + \frac{45}{45}$

$\int_0^1 (x^4-1)^2\,dx = \frac{5}{45} - \frac{18}{45} + \frac{45}{45} = \frac{32}{45}$

b) $\int_1^9 \left(\frac{1}{2x}\right)dx = \frac{1}{2}\int_1^9 \frac{1}{x}\,dx = \frac{1}{2}\left[\ln(x)\right]_1^9 = \frac{1}{2}\left(\ln(9)-\ln(1)\right)$

$\int_1^9 \left(\frac{1}{2x}\right)dx = \frac{1}{2}\left(\ln(9)-0\right) = \frac{\ln(9)}{2}$ ✓

c) $\int_1^2 \frac{4+x^2}{x}\,dx = \int_1^2 \left(\frac{4}{x}+\frac{x^2}{x}\right)dx = \int_1^2 \frac{4}{x}\,dx + \int_1^2 x\,dx$ ✓

$= 4\int_1^2 \frac{1}{x}\,dx + \int_1^2 x\,dx = 4\left[\ln(x)\right]_1^2 + \left[\frac{x^2}{2}\right]_1^2 = 4(\ln(2)-0) + \left(\frac{4}{2}-\frac{1}{2}\right)$

$\int_1^2 \frac{4+x^2}{x}\,dx = 4\ln(2) + \frac{3}{2}$ ✓

d) $\int_0^1 \frac{e^{3x}+1}{e^x}\,dx = \int_0^1 \frac{e^{3x}}{e^x}\,dx + \int_0^1 \frac{1}{e^x}\,dx = \int_0^1 e^{2x}\,dx + \int_0^1 e^{-x}\,dx$

$= \left[\frac{1}{2}e^{2x}\right]_0^1 + \left[-e^{-x}\right]_0^1 = \left[\frac{1}{2}e^2 - \frac{1}{2}\right] + \left[-e^{-1}+1\right]$

$\int_0^1 \frac{e^{3x}+1}{e^x}\,dx = \frac{1}{2}e^2 - \frac{1}{e} + \frac{1}{2}$